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ELEMENTS OF ARITHMETIC. PART SECOND.

THE

GRAMMAR-SCHOOL

ARITHMETIC:

[illegible]

VALUABLE COMMERCIAL INFORMATION;

[illegible]

[illegible]

BY

HORACE MANN, LL.D.,

[illegible]

AND

PLINY E. CHASE, A.M.

[illegible]

PHILADELPHIA:
PUBLISHED BY E. H. BUTLER & CO.
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SIGILLVM ACADEMIÆ HARVARDIANÆ IN NOV ANG
CHRISTO ET ECCLESIÆ

The Gift of
Pliny Earle Chase
of
Philadelphia
(Class of 1839)
28 July 1858

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ELEMENTS OF ARITHMETIC, PART SECOND.

THE

GRAMMAR-SCHOOL ARITHMETIC:

CONTAINING MUCH

VALUABLE COMMERCIAL INFORMATION;

TOGETHER

WITH A SYSTEM OF INTEGRAL, DECIMAL, AND FRACTIONAL ARITHMETIC, SO ARRANGED
AS TO DISPENSE WITH MANY OF THE ORDINARY RULES.

BY

HORACE MANN, LL.D.,

THE FIRST SECRETARY OF THE MASSACHUSETTS BOARD OF EDUCATION.

Earle AND

PLINY E. CHASE, A.M.,

AUTHOR OF THE COMMON-SCHOOL ARITHMETIC.

PHILADELPHIA:
PUBLISHED BY E. H. BUTLER & CO.
1855.

(1851)

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~~118.515~~ Educ T 118.55.550

1858 July 20
Gift of
Pliny Earle Chase
of Philadelphia
Class of 1839

Entered, according to the Act of Congress, in the year 1851, by
HORACE MANN AND PLINY E. CHASE,
in the Clerk's Office of the District Court of the United States, in and for
the Eastern District of Pennsylvania.

PREFACE.

Our "Primary-School Arithmetic," the "Common-School Arithmetic," and our Sequel, or "Arithmetic Practically Applied," having been previously published, the present work completes a series. This series we have endeavored to arrange in such a manner, as to meet the wants of schools of every grade, and to stimulate while it rewards the exertions of scholars.

For schools where only a *single work* is desired for all the classes, it is believed that the Common-School Arithmetic will be found more comprehensive and thorough than any other work of the same size.

Where there are two grades of schools, the Primary Arithmetic may be used with advantage in the lower grade, and either the Grammar-School Arithmetic, or the Common-School Arithmetic, in the higher.

In towns where there are three or four grades of schools, and three or more different books are required, either the Grammar-School Arithmetic, or the Arithmetic Practically Applied, should be used for revision and completeness. The former occupies nearly the ground of the higher class of Arithmetics ordinarily used in our common schools; the latter tests the thoroughness of all who may study it, facilitates the application of principles to almost every kind of employment, and opens the eye to behold many of the infinite wonders of science.

In the present work, the pupil is supposed to be thoroughly acquainted with our Primary-School Arithmetic; and scholars should not study this, who have not an *a-b-c* familiarity with the operations of all the simple rules upon Integers, Decimals, and Common Fractions.

The "Examples for the Board" are designed for the guidance of the Teacher in his oral Explanations. When entering upon any new subject, he should not only illustrate its processes to the class, as clearly as possible, but he should lay open the connection between cause and effect, so that the pupil may see that the consequent, in all cases, flows from the antecedent, by a logical necessity, and that one of them could not be different from what it is, without a corresponding difference in the other. Much of the Scholar's ultimate success will depend upon the Teacher's early faithfulness in this particular.

In the following work, we have given numerous abstract examples under all the more important rules. These will furnish means of practice. Repetition, under various forms, not only impresses the memory, but gives a clearer and more abiding comprehension of principles. We have taken pains to intersperse many questions of interest and curiosity throughout the work, hoping thereby to render it attractive as well as instructive. We have also retained the novel features of the previous editions of the Elements of Arithmetic, prominent among which are the following:

1. Integers, Decimals, and Federal Money, are taught in connection. This is an arrangement founded on Principle, and not on Form. Instead of separating, it associates subjects which have a natural affinity, and it supersedes the various and perplexing rules for Decimals and Federal Money, which encumber other works. The pupil naturally regards every new rule as the development of a new principle, and he will not readily discover

any connection between subjects that are treated in different parts of his text-book, and under distinct heads. Taught to look at the form rather than the substance, he will not see that oneness of principle, which gives us so many less things to remember, and helps us to remember whatever it gives.

The unanimous testimony in favor of this arrangement by all the distinguished teachers who have tried it, gives the authority of successful experiment to the conclusions of *à priori* reasoning.

2. Throughout the work, the rules have been generalized by referring the several branches or subdivisions of a subject to a few general principles. By this means, many of the rules usually given are dispensed with, and many that are retained are greatly simplified in form. For example; after the scholar has learned to add, subtract, multiply, and divide fractions and decimals, there is no occasion for new and separate rules for fractions and decimals, in Compound Numbers, Percentage, Proportion, &c. In like manner, after having learned that a fraction is merely the representative of a division, the simple intimation that a fraction may be reduced to a whole or mixed number, or to a decimal, by merely PERFORMING *the division that the fraction* EXPRESSES, will be more easily remembered than three formal rules for the several operations.

3. We have introduced much valuable information that is generally omitted, but which is continually required in the counting-house. For instances of this kind, we refer particularly to the Sections on Compound Equation and Percentage of Sterling Money, and to the Commercial Dictionary and Tables.

4. Under many of the rules, new applications are given, that we believe will be found both interesting and valuable. For some of these, see the Articles on Fellowship and Proportion. In the treatment of the latter subject, we have taken especial

pains to show the connection of the proportional statement with ordinary analysis, and to remove the impression that the "Rule of Three" is altogether arbitrary, by showing a good and sufficient reason for the arrangement of terms that it requires.

Our aim has been to prepare a series that would meet the approval of all teachers who consider thorough scholarship more important than working by rote. We could have treated many subjects more simply than we have done; we could have offered books that would have required less labor on the part of the instructor and the pupil; but believing that whatever is worth learning at all, is worth learning well, we have been unwilling to lend our sanction to indolence and superficial teaching.

HORACE MANN,
PLINY E. CHASE.

May, 1851.

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ARITHMETIC.

I. DEFINITIONS.

1. ARITHMETIC is the art of computing by numbers.

A NUMBER, in Arithmetic, is an expression that denotes how many are spoken of.

Written numbers may be expressed by words, by letters, or by combining the ten Arabic figures.

One, Two, Three, Four, Five, Six, Seven, Eight, Nine, Naught.
1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

The first nine of these characters are called *digits*, from the Latin *digitus*, a finger; because the numbers they express are often counted on the fingers.

The tenth character, 0, expresses no value by itself, and is therefore commonly called *Naught*. It is also called *Zero* or *Cipher*.

The number One is also called a *Unit*, and it serves as a standard for comparing all numbers of a kind. Thus when we speak of ten miles, twenty houses, fourteen men, or sixty bushels, we are supposed already to have a clear idea of one mile, one house, one man, or one bushel, which is assumed as the unit.

All numbers may be considered as derived from the Unit; for one and one are two; two and one are three; three and one are four; and so on to any extent we please.

An **ABSTRACT NUMBER**, is a simple number, which has no reference to any thing in particular; as *five, seventeen, six.* An **APPLICATE**, or **CONCRETE NUMBER**, is a number applied to some particular thing or things; as *five books, seventeen days, six men.*

An **INTEGER** is a whole number; as ten, twenty-nine, forty-five. A **FRACTION** is a part or parts of an integer; as one-half, two-thirds, seven-tenths.

Arithmetic embraces five fundamental operations, in each of which something is given, and something is required; namely, Numeration, Addition, Subtraction, Multiplication, and Division.

In Numeration, numbers are either given in figures to be read in words, or given in words to be written in figures.

In Addition, two or more numbers are given, and their sum or amount is required.

In Subtraction, two numbers are given, and their difference is required.

In Multiplication, two numbers are given, and their product is required.

In Division, two numbers are given, and their quotient is required.

II. NUMERATION.

2. **NUMERATION** is the process by which we express numbers in letters or figures, and read them when so expressed.

There are two modes of numeration now in use: the **ROMAN** and the **ARABIC**.

3. Roman Notation.*

The Roman method employs the following letters:

One,	Five,	Ten,	Fifty,	One hundred,	Five hundred,	One thousand.
I,	V,	X,	L,	C,	D,	M.

When a letter stands immediately before another which denotes a larger number, the value of the first must be taken from that of the second. In all other cases, the values of the different letters must be added together, as in the following table:

I.	is	One.	XX.	is	Twenty.
II.	"	Two.	XXI, &c.	"	Twenty-one.
III.	"	Three.	XXX.	"	Thirty.
IV.	"	Four.	XL.	"	Forty.
V.	"	Five.	L.	"	Fifty.
VI.	"	Six.	LX.	"	Sixty.
VII.	"	Seven.	LXX.	"	Seventy.
VIII.	"	Eight.	LXXX.	"	Eighty.
IX.	"	Nine.	XC.	"	Ninety.
X.	"	Ten.	C.	"	One hundred.
XI.	"	Eleven.	CC, &c.	"	Two hundred.
XII.	"	Twelve.	D.	"	Five hundred.
XIII.	"	Thirteen.	DC, &c.	"	Six hundred.
XIV.	"	Fourteen.	M.	"	One thousand.
XV.	"	Fifteen.	MC.	"	One thousand one hundred.
XVI.	"	Sixteen.	MDCCCC.	"	One thousand nine hundred.
XVII.	"	Seventeen.	MM.	"	Two thousand.
XVIII.	"	Eighteen.	$\overline{\mathrm{C}}$.†	"	One hundred thousand.
XIX.	"	Nineteen.	$\overline{\mathrm{M}}$.	"	One thousand thousand.

4. Arabic Notation.

The Arabic method is much more convenient, and is employed in all the ordinary operations of Arithmetic. By this method, *every figure is made to represent a value ten times as large by each removal to the left, and one-tenth as large by each removal to the right.* Thus if 1 represents one,

---

* That part of Numeration which treats of the expression of numbers by letters or figures, is sometimes called Notation.

† A dash placed over any letter, increases its value a thousand times.

10 will represent ten; 100, ten tens, or one hundred; 20, two tens, or twenty; 40, four tens, or forty; &c.

A point (.) called the *decimal point*, or *separatrix*, is placed at the right of units, to distinguish them from parts of units or *decimals*. The first figure at the left of the separatrix denotes units, the second tens, the third hundreds. These three are usually embraced under the name of units; the next three figures embrace units, tens, and hundreds, of thousands; the next, units, tens, and hundreds, of millions, &c., as in the following table:

Duodecillions.	Undecillions.	Decillions.	Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
54	123	006	862	040	000	000	016	500	000	000	479	513	660*

The places at the right of units are called tenths, hundredths, thousandths, &c., (denoting that the unit is divided into 10, 100, 1000, &c., parts,) as in the following table:

TRILLIONS.	Hundred billions.	Ten billions.	BILLIONS.	Hundred millions.	Ten millions.	MILLIONS.	Hundred thousands.	Ten thousands.	THOUSANDS.	Hundreds.	Tens.	UNITS.	Tenths.	Hundredths.	THOUSANDTHS.	Ten thousandths.	Hundred thousandths.	MILLIONTHS.	Ten millionths.	Hundred millionths.	BILLIONTHS.	Ten billionths.	Hundred billionths.	TRILLIONTHS.
6,	5	0	0,	0	5	7,	0	0	6,	9	0	8.	8	1	6	9	0	0	7	4	0	5	2	1

In reading decimals, *read first the whole number, (if any,) then read the decimal as if it expressed a whole num-*

---

* The English formerly divided numbers into periods of six figures each, calling a million million a billion, a million billion a trillion, &c. The division into periods of three figures, which is called the French method, is now generally adopted.

ber, assigning it the value of the right hand decimal place. Thus 29000001.004063, is read, 29 million and 1, and 4 thousand and sixty-three *millionths.*

5. EXAMPLES FOR THE BOARD.[*]

.0000090864
16.0003259
2987.360042
4.000000089443

6. EXAMPLES FOR THE PUPIL.

1. One sheep fly will produce 20000 young, and the larvæ in 24 hours increase their weight 200 fold.

2. The velocity of sound in sea water, is 4708 feet per second, at the mean temperature of 46.6° Fahrenheit.[†]

3. Prof. Ehrenberg has shown the existence of animals in fluids, of which each drop contains 500000000.[‡]

4. A Sanscrit Almanac, published in 1670, stated the age of the world at 3892771 years from its last creation.

5. The quantity of wood consumed in one year on 560 miles of railroad in Massachusetts, has been estimated at 53710 cords, worth $178990.[§]

6. Estimating the diameter of the earth at 7930 miles, its solid contents would be 38434476263828705280000 cubic feet.

7. In the fourth crop from a single seed of the hyoscyamus, there would be nearly 6250000000000000000 plants, provided each seed should produce one of the most fertile plants.

8. If a cent were kept at compound interest, so as to

---

[*] The Examples for the Board should be solved by the Teacher, in the presence of the class, and fully explained.

[†] Pierce. [‡] McGauley. [§] G. B. Emerson.

double every 11 years, it would amount in 517 years to $1407374883553.28.

9. The reputed inventor of the game of chess, is said to have asked a reward which would be equivalent to 31329388712142.58 bushels of wheat.

10. .08
11. .8
12. .008
13. .13876
14. .000205
15. 13.054
16. 208.0010076
17. 179002.0037
18. 37000005830.0000870654
19. 2908540002.000000000000001
20. 1.000000800000070000000000000000001
21. 26501.00000000000080567809120566
22. 9003075200000854212.0000000005001376
23. 116840000000000000000000000000000.05
24. 641000000873004000500926050387000.596
25. 5000000000000000000005000000005000005.0005

26. In 1824 Carlini found the attraction of Mt. Cenis to produce a variation in the oscillations of a pendulum, to correct which it was necessary to lengthen it .0082677 of an inch.

27. The thickness of a soap bubble when the colors are first seen in it, is about .000003937 of an inch.

28. The circumference of a circle may be found very nearly, by multiplying the diameter by 3.14159265358979323846 26433832795028841971693993751058209749445923.

7. Decimal Notation.

Refer to the table of decimals, and observe that

3	decimal	places	represent	thousandths.
6	"	"	"	millionths.
9	"	"	"	billionths.
12	"	"	"	trillionths.
15	"	"	"	quadrillionths.
18	"	"	"	quintillionths.
21	"	"	"	sextillionths.
24	"	"	"	septillionths.
27	"	"	"	octillionths.
30	"	"	"	nonillionths.
33	"	"	"	decillionths.
36	"	"	"	undecillionths.
39	"	"	"	duodecillionths.

If it were required to write five thousand and eight *hundred billionths*, we should first write 5008. Billionths requiring nine decimal places, hundred billionths will require two more, or eleven. As 5008 has but four figures, seven zeroes must be prefixed to make the required number of places, thus, .00000005008. If this be numerated it will be found to express *hundred billionths*, as required.

Rule for writing Decimals. *Write first the whole numbers, (if any,) and put the decimal point at their right. Then write the decimals like whole numbers, and, if necessary, prefix zeroes to make up the required number of decimal places.*

8. EXAMPLES FOR THE BOARD.

115 septillion, 2 trillion, 97 thousand and 5—and 49 thousand and 3 *hundred thousandths.*

Change the decimal in the above example successively to *trillionths*, *hundred quadrillionths*, and *ten sextillionths.*

19 billion 3 million 1 thousand and 16 *ten millionths* is equivalent to what whole number?

9. EXAMPLES TO BE WRITTEN, AND PROVED BY NUMERATING.

1. The mean distance of the moon from the earth is about two hundred and forty thousand miles.

2. The mean distance of the earth from the sun, is about ninety-five million miles.

3. The circumference of the earth at the equator, is about twenty-four thousand eight hundred and ninety-five and five-tenths miles.*

4. The highest known mountain on the face of the globe is Dhawalagir, a peak of the Himalaya range, which has been estimated by one measurement at twenty-six thousand eight hundred and seventy-two feet; by another, at twenty-eight thousand and fifteen feet in height.†

5. In six thousand years there are only one hundred and eighty-nine billion three hundred and forty-one million three hundred and thirty-eight thousand and two hundred seconds. A trillion seconds would be equivalent to more than thirty-one thousand six hundred and eighty-eight years.

6. The population of the globe was estimated in the Weimar Almanac of 1840, at nine hundred and ninety-three million ninety-five thousand eight hundred and seventeen.

7. The period of our sun's revolution around the central sun, is estimated by Maedler at eighteen million two hundred thousand years.‡

8. It is estimated that some of the stars which are visible

---

* Downes. † Encyclopædia Americana. ‡ Nichol.

through the telescope, are so remote that their light, traveling at the rate of one hundred and ninety-two thousand miles per second, does not reach us until the end of thirty million years.*

9. The population of China in 1825, according to Dr. Morrison, was three hundred and fifty-two million eight hundred and sixty-six thousand and two.

10. The entire weight of our atmosphere has been estimated at eleven quintillion eight hundred and fifty three quadrillion six hundred and sixty-four trillion pounds.

11. The amount of the British National Debt in 1847, estimating the pound sterling at four dollars and eighty-four cents, was three billion seven hundred million seven hundred and four thousand and ninety-four dollars and fifty-six cents.†

12. Ten thousand and three,—and fifty-six *thousandths.*

13. Two hundred and nine,—and fifteen million and seven *trillionths.*

[The remaining examples in this section may be left for the review, if they are found to be too difficult for the first course.]

14. Six quadrillion seventeen billion and twenty-four,—and six quadrillion seventeen billion and twenty-four *hundred-sextillionths.*

15. Seventy-five septillion three quadrillion four hundred and one thousand,—and four octillion eleven thousand and six *ten-decillionths.*

16. Two quadrillion thirty trillion four hundred million and five,—and six decillion and seven *duodecillionths.*

17. One hundred duodecillion,—and one *hundred-duodecillionth.*

18. One hundred and fifty sextillion fifty-one quintillion

---

* Nichol.

† Annual Register.

and thirty-six trillion,—and six hundred and thirty billion and five *sextillionths.*

19. Twenty-eight nonillion forty-six octillion seven sextillion and nine hundred thousand,—and five hundred and twenty quintillion forty thousand and seven hundred and eight *ten-septillionths.*

20. Seventeen undecillion thirty-one trillion two hundred and fifty billion,—and ninety-five thousand and thirteen *hundred-quintillionths.*

21. Nine hundred and ninety-nine *undecillionths.*

22. Nine hundred,—and ninety-nine *undecillionths.*

23. Five hundred and five undecillion seventy-six nonillion and three quintillion,—and fourteen million and fifty-nine *ten-octillionths.*

24. Six duodecillion twenty-two quadrillion seven hundred and seventy-seven *hundred-duodecillionths.*

It has already been stated that zeroes have no value of themselves, their only use being to supply the place of denominations that are wanting. *Any number of zeroes, therefore, placed at the right hand of decimals, does not alter their value.*

III. ADDITION.

10. ADDITION is the process by which we join two or more numbers together to find their *sum* or *amount.*

The sign + (plus) signifies that two or more numbers are to be added together. The sign = (equal to,) denotes that one number or series of numbers is equal to another. Thus $2+3+5=10$, signifies that the sum of 2, 3, and 5 is equal to 10.

11. Example for the Board.—Add together 196, 73.22, 18.09, 614.4, and 1.96.

Hundreds.	Tens.	Units.	Tenths.	Hundredths.
1	9	6.		
	7	3.	2	2
	1	8.	0	9
6	1	4.	4	
		1.	9	6
9	0	3.	6	7

The sum of the column of hundredths is 17 hundredths, or 1 tenth and 7 hundredths. It would evidently be wrong to write tenths beneath the column of hundredths. We therefore only write down the 7 hundredths, and add, or *carry*, the 1 tenth to the column of tenths. In adding the tenths, we say 9 and 1 *to carry* are 10, and 4 are 14, and 2 are 16. We have here 1 to carry to the units' column,—and in continuing our work, we find we have 2 to carry to tens, and 2 to hundreds.

Teach the pupil to *prove* his work by adding downwards, and show the various other methods of proof.

12. Federal Money, or the currency of the United States, is written like decimals, the dollar, [$], being regarded as the unit, the cents as hundredths, and the mills as thousandths.

13. EXAMPLES FOR THE PUPIL.

1. Add five thousand and thirty-one dollars; two thousand eight hundred dollars and six cents; nine hundred dollars and eleven cents; five dollars sixty-two cents and five mills; and three hundred and fifty dollars and five mills. *Ans.* 9086.80.

2. Find the sum of 960840.276 + 28890037.0009 + 5613.02 + 2988135.921 + 60. *Ans.* 82844686.2179.

3. Find the sum of 285.9; 14.283; 1390.0025; 268; 7412.09; 3.846; 176; 506.000007; 87.284003; 441.929; and 3765.0761.

4. A steamer on the first nine days of her voyage sailed as follows: 308.02 miles; 295.0009 miles; 315.769 miles; 301.0035 miles; 256 miles; 261.4 miles; 279.0908 miles; 300.96 miles; and 298.239 miles. How far had she gone at the end of the ninth day?

5 Ten men entered into partnership, contributing as follows: $1487.638; $1960.00; $1276.45; $1195.10; $2004.375; $1703.199; $751.25; $3475.871; $11240.50; and $989.112. What amount was invested in the firm?

6. 448.771 + 2297.08 + 13596 + 67.0954 + .008876 + 14.32094 + 1559.198874 + 420 + 5995682 + .4830611 + 25700 + 619.098 + 477122.6003 = ?

Ans. 6517526.6564511.

7. Add together 197446.00887932; 466108.4434097; 831165; 97061588.4003; 5425980.4991844; 761716604.00491883; 26915224.20211; 8112907690080; 5176448.037165222; and 9084129765.

Ans. 8122889110409.595967472.

8. Required the population of the New England States, according to the census of 1850, the population of Maine being 583026; New Hampshire, 317997; Vermont, 314332; Massachusetts, 996665; Rhode Island, 147503; Connecticut, 371982.

9. Find the population of the Middle States, according to the census of 1850, the population of New York being 3099249; New Jersey, 490070; Pennsylvania, 2326208; Delaware, 90407; Maryland, 583016; District of Columbia, 54000.

10. What was the population of the South-Eastern States, according to the census of 1850, the population of Virginia being 1428863; North Carolina, 870509; South Carolina, 639099; Georgia, 950000; Florida, 85000?

11. The population of Alabama, according to the census of 1850, was 820060; Mississippi, 580000; Louisiana, 525000; Texas, 175000; California, 165000; Arkansas, 198776; Tennessee, 1069000. What was the entire population of the South-Western States?

12. According to the census of 1850, the population of Missouri was 681547; Kentucky, 993344; Ohio, 1981980; Indiana, 990258; Illinois, 850000; Michigan, 400000; Wisconsin, 805121; Iowa, 192974. What was the entire population of the North-Western States? Of the United States? *Ans. to the last*, 23387125.

IV. SUBTRACTION.

14. Subtraction is the process by which we take one number from another, to find their *difference*.

When one number is subtracted from another, the larger number is said to be *diminished* by the smaller. The number *to be diminished*, is the *minuend*. The number *to be subtracted*, is the *subtrahend*. The number obtained by subtraction is the *difference*, the *remainder*, or the *excess* of the larger over the smaller number.

The sign — (minus,) signifies that the latter of two numbers is to be taken from the former. Thus 7 — 5 = 2, signifies that if 5 be taken from 7, the remainder will be 2.

15. EXAMPLE FOR THE BOARD.

Subtract 187.81 from 329.32.

Hundreds.	Tens.	Units.		Tenths.	Hundredths.
3	2	9	.	3	2
1	8	7	.	8	1
1	4	1	.	5	1

Subtracting in the usual way, we cannot take 8 tenths from 3 tenths. It may readily be shown, that if 10 be added to two numbers their difference will be the same. This is equally true whether we add tenunits, ten tenths, or ten of any other denomination. If we then add 10 tenths to the 3, and its equivalent, 1 unit, to the next figure we subtract, the difference will be the same. Then, *whenever any figure of the minuend is smaller than the figure below it in the subtrahend, we may add* 10 *to the upper figure, and carry* 1 *to the next figure we subtract.* Proceeding in this way, we

say, 8 tenths from 13 tenths leave 5 tenths; 8 units from 9 units leave 1 unit; 8 tens from 12 tens leave 4 tens, and 2 hundreds from 3 hundreds leave 1 hundred.

Teach the pupil to *prove* his work, by adding the subtrahend and remainder.

16. EXAMPLES FOR THE PUPIL.

1. What is the difference between $88463.11 and $79521.097 ?

2. The population of the United States in 1830 was 12858670. In 1840 it was 17068666. What was the increase in ten years?

3. A gentleman has invested $140768.25 in two estates, one of which is worth $89329.18. What was the cost of the other? *Ans.* $51439.07.

4. A. said to B., I am worth $116205.39. B. said, I am not worth so much, by $49164.42. What was B. worth? *Ans.* $67040.97.

5. 8843900269517 — 2988143675.0087906 = ?
Ans. 8840912125841.9912094.

6. What is the excess of $14943870.01 over $9568841.095? *Ans.* $5375028.915.

7. A broker purchased stock for which he paid $259084.63. Did he gain or lose, by selling the same stock for $290451.00? How much?

8. In a certain factory there were made 916482.7 yards of satinet, and 895267.594207 yards of broadcloth. How many yards were there of satinet more than of broadcloth?
Ans. 21215.105793 yds.

9. The population of the United States in 1850

was 23387125. What was the increase in the preceding ten years? In the preceding twenty years?

10. The population of the United States in 1790 was 3929827. What was the increase in sixty years?

11. The population of the United States in 1800 was 5305925; in 1810, 7239814; in 1820, 9638131. What was the increase from 1790 to 1820? from 1800 to 1850?

V. MULTIPLICATION.

17. MULTIPLICATION is the process by which we find the sum of a number or part of a number, when repeated a given number of times.

The number *to be multiplied* or repeated, is the multiplicand. The number *to multiply by*, or the number of times the multiplicand is repeated, is the *multiplier*. The sum obtained by multiplication, is the *product*. The multiplier and multiplicand are also called *factors* of the product. If there are more than two factors, we first obtain the product of any two, then multiply this product by a third, and so on. The final product is called the *continued product* of all the factors. Thus 4 times 3 times 5 times 2 = 120.

Any number that can be resolved into factors, is a *composite number*. A number that cannot be so resolved, is a *prime number*. Thus 6, 12, 16, &c., are composite numbers; 1, 2, 3, 5, 7, 11, &c., are prime numbers.

The sign × (times,) denotes that one number is to be multiplied by another, as, $4 \times 3 = 12$; $2 \times 6 \times 3 = 4 \times 9$.

18. EXAMPLES FOR THE BOARD.

If a barrel of flour costs $4.75, what will 4 barrels cost?

(1.)
4.75
4.75
4.75
4.75
19.00

(2.)
4.75 *Multiplicand.*
4 *Multiplier.*
19.00 *Product.*

The answer may be obtained as in (1), by addition. But as the same figures are repeated 4 times, the sum of each column may be readily found by multiplication, as in (2). We say, 4 times 5 hundredths are 20 hundredths, or 2 tenths and 0 hundredths; 4 times 7 tenths are 28 tenths, and 2 tenths to carry are 30 tenths, or 3 units and 0 tenths; 4 times 4 units are 16 units, and, 3 units to carry are 19 units.

If a barrel of flour costs $4.88, what will 15 barrels cost?

4.88
15
24.40
48.8
73.20

The product of 4.88 by 5 units, is 24.40. In multiplying by 1 ten, the product will be 10 times as large as the product by 1 unit, and each figure will therefore stand 1 place farther to the *left*. The product by 1 ten is then 48.8, and the sum of the two products, or the product by 15, is 73.20.

If a barrel of flour costs $4.88, what will 15.5 barrels cost?

4.88
15.5
2.440
24.40
48.8
75.640

The units figure of the multiplier is written under the right-hand figure of the multiplicand, because the product of any denomination by units is of the same denomination; *i. e.* units by units give units, tenths by units give tenths, and so on. The product by tenths is 10 times smaller than the product by units, and each figure will therefore stand 1 place farther to the *right*. Therefore in preparing any numbers for multiplication, *place the units' figure of the multiplier under the right-hand figure of the multiplicand, and place the first figure of each product under the figure by which you multiply. The separatrix of each product will then fall under that of the multiplicand.*

A careful examination of this process will show, that *there are always as many decimals in the product as there are in both the multiplier and the multiplicand.*

19. *The product by zero is always zero.*

To multiply any number by 10, 100, 1000, &c., *remove the decimal point as many places to the right, as there are zeroes in the multiplier.*

When there are zeroes at the right hand of either, or both factors, *perform the multiplication without them, and annex them to the product.*

If the multiplier can be resolved into factors, *the product may be obtained either by employing the whole multiplier, or each of its factors in succession.*

20. EXAMPLES FOR THE PUPIL.

1. The height of Mount Etna, as trigonometrically determined by Capt. W. H. Smyth, in 1814, was 10874 feet; as determined by Sir John Herschel in 1824, with the barometer, 10872.5 feet.* Required the result of each measurement in inches.

2. 49863 × .06 × 1.8 = ? *Ans.* 3889.314.

3. Multiply 7441.08 by 236. *Ans.* 1756094.88.

4. Find .06 of 284931.75.

5. Find .0725 of 164.25 × 1000. *Ans.* 11908.125.

6. The volume of water discharged by the Nile at its flood, has been estimated at 8166.065 cubic metres per second.* To what number of gallons is this equivalent, a cubic metre being 264.19 gallons?

7. 96.8 × 12000 × 45.93 = ? *Ans.* 53352288.

8. Multiply 4489.076 by 2604000.
Ans. 11689553904.

9. Multiply 596000 by 70490000.

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* Journal of the Royal Geographical Society, 1850.

10 What is the product of $36 \times 24 \times 12 \times 15$?

Ans. 155520.

11. A merchant borrowed $9965.73, and paid .1845 of the amount for the use of the money. How much did he pay in the whole? *Ans.* $11804.407.

12. Multiply 909871265000 by 470.0368.

Ans. 427672977812552.

VI. DIVISION.

21. DIVISION is the process by which we find how many times one number or part of a number is contained in, or may be subtracted from another.

The number *to be divided* is the *dividend.* The number *to divide by*, is the *divisor.* The number of times the dividend contains the divisor, is the *quotient.* The divisor and quotient may also be regarded as factors of the dividend. The number left, (if any,) after the operation, is the *remainder.*

The sign ÷[*] (divided by) signifies that the former of two numbers is to be divided by the latter, as; $4 \div 2 = 2$; $16 \div 4 = 8 \div 2$. Division may also be expressed by writing the divisor under the dividend; as, $\frac{4}{2}$, $\frac{16}{4}$, $\frac{7}{8}$, $\frac{11}{12}$; which are read $4 \div 2$ or 4 *halves;* $16 \div 4$ or 16 *fourths;* $7 \div 8$ or 7 *eighths;* $11 \div 12$ or 11 *twelfths*, &c. Numbers written in this manner are called *fractions*, the number above the line, or the dividend, being the *numerator*, and the number below the line, or the divisor, the *denominator.* The remainder in any division may always be written as the numerator of a fraction, whose denominator will be the divisor.

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[*] The two points indicate the places which the dividend and divisor would occupy, if the division were represented in a fractional form.

22. EXAMPLE FOR THE BOARD.

Divide 36.3 by .0017.

It has already been stated that the divisor and quotient may be regarded as factors of the dividend. It follows, therefore, from the principles of multiplication, that the dividend must contain as many decimals as both divisor and quotient, and that the quotient must contain as many decimals as there are in the dividend, more than in the divisor. If the dividend has fewer decimals than the divisor, we must annex decimal naughts until it has as many or more than the divisor. In the present instance we must annex at least three naughts to the dividend, and we may add more if we choose.

$.0017)36.3000(21352\frac{16}{17}$
34
23
17
60
51
90
85
50
34
16

Commencing the division, we find that 17 is contained in 36, 2 times. We place 2 in the quotient, and subtract 2 × 17 from 36. The remainder is 2.3. 17 is contained in 23, 1 time. Place 1 in the quotient, and subtract 1 × 17 from 23. To the remainder 6 we annex one of the 0s, and find that 17 is contained in 60, 3 times with 9 remainder. We continue this process, annexing to each remainder a new figure of the dividend, until we find a final remainder 16, which does not contain 17, but the division by 17 may be expressed by writing the divisor underneath.

If we desire decimals in the answer, we must continue to annex decimal 0s until we have obtained the desired number.

Always have at least as many decimal places in the dividend as there are in the divisor. Point off as many decimals in the quotient as there are in the dividend more than in the divisor.

23. To divide any number by 10, 100, 1000, *&c., remove the decimal point as many places to the left as there are zeroes in the divisor.*

When there are zeroes at the right hand of the divisor, *cut them off, and remove the decimal point of the dividend as many places to the left. After dividing the integers, remove the decimal point from the remainder, and you will have the true remainder.*

When the divisor can be resolved into factors, *we may either employ the whole divisor, or each of its factors in succession.*

When the quotient will contain a number of figures, *it is often convenient to multiply the divisor by each of the nine digits, and write the products on the slate, before commencing the division. It will then be easy to determine the value of each quotient figure, and the product to be subtracted from the partial dividend.*

24. EXAMPLES FOR THE PUPIL.

1. Determine which of the following numbers are composite, and resolve all the composite numbers into their prime factors,* 327; 125; 512; 149; 343; 137; 216; 240; 420; 30030.

2. Divide .0497 by 368000. *Ans.* .000000135.

3. What is the quotient of 3.995 by $2 \times 3 \times 4$? *Ans.* .166458.

4. $8.9 \times .3 \times 14 \div (27 \times .04 \times 13) = ?$ *Ans.* 2.6623.

5. If the product of two factors is 81.45, and one of the factors is 18.1, what is the other?

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* Any number may be resolved into its prime factors, by dividing by the prime numbers in regular order, employing each number as a divisor, as many times as it will go without a remainder. Thus in the example in the margin, we find that $252 = 2 \times 2 \times 3 \times 3 \times 7$.

2	252
2	126
3	63
3	21
7	7
	1

6. If the product of four factors is 2520, and three of its factors are 8, 5, and 7, what is the fourth ?

7. Divide 4 × 7 × 9 × 2 by 9 × 7 × 2. By 9 × 4. By 4 × 2 × 9.

8. Divide .00984 by .02613 × 7.

9. What is the quotient of .087 × .003 by 19000 × 70?

Ans. .00000000019624.

10. 27.3 is .16 of* what number? .0095 is .84 of what number? 76.125 is 1.25 times what number? 841.21 is 1.34 of what number? 44 is .13 of what number?

25. The Least Common Multiple.

One number is called a *multiple* of another, when the former can be divided by the latter without any remainder. A number that can be exactly divided by two or more other numbers is a *common multiple* of those numbers. The divisors are called *sub-multiples,* or *aliquot parts.* Thus 24 is a common multiple of 2, 4, 6, 8, and 12,—and these latter numbers are *aliquot parts* of 24. Any number that contains all the factors of a number, will evidently contain the number itself. Thus 24, which is equal to 2 × 2 × 2 × 3, contains 2 × 2 or 4, 2 × 3 or 6, 2 × 2 × 2 or 8, and 2 × 2 × 3 or 12. Then the least common multiple of any series of numbers, is the least number which contains all the factors of the given numbers, and may be found by the following

RULE.

Arrange the numbers in a horizontal line, and divide successively by the prime numbers, 2, 3, 5, 7, 11, *&c., employing each divisor as often as it will divide one or more*

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* Show the pupil that for the word "of," when used in this connection, the word "times" may be substituted.

of the numbers without a remainder, writing the quotients and undivided numbers, beneath. Continue this division until the last quotients are all 1, *and you will have obtained all the prime factors of the given numbers. The product of these factors, is the least common multiple.*

EXAMPLE FOR THE BOARD.

Find the least common multiple of 14, 18, 27, 21, 28, 126. Dividing by the rule, we find that the only prime factors contained in the given numbers are 2, 2, 3, 3, 3, and 7. Their product is 756, which is the least common multiple.

2)14 “ 18 “ 27 “ 21 “ 28 “ 126
2)7 “ 9 “ 27 “ 21 “ 14 “ 63
3)7 “ 9 “ 27 “ 21 “ 7 “ 63
3)7 “ 3 “ 9 “ 7 “ 7 “ 21
3)7 “ 1 “ 3 “ 7 “ 7 “ 7
7)7 “ 1 “ 1 “ 7 “ 7 “ 7
1 “ 1 “ 1 “ 1 “ 1 “ 1

By resolving the numbers into their prime factors, or by the table of prime factors, the least common multiple can sometimes be found more readily, in the following manner:

Form the product of all the prime factors of the given numbers, employing each factor the largest number of times it is used in either number.

Referring to the numbers in the foregoing example, we find they are respectively equal to 2×7, $2 \times 3 \times 3$, $3 \times 3 \times 3$, 3×7, $2 \times 2 \times 7$, and $2 \times 3 \times 3 \times 7$. The only prime numbers used, are 2, 3, and 7. 2 is employed 2 times in the 5th number, 3 is employed 3 times in the 3d number, and 7 is employed but once in either number. The least common multiple is then, $2 \times 2 \times 3 \times 3 \times 3 \times 7 = 756$, as before.

26. EXAMPLES FOR THE PUPIL.

1. Find the least common multiple of 96, 32, 12, 24, and 48.

2. Find the least common multiple of 32, 27, 16, 18, and 54. *Ans.* 864.

3. Find the least common multiple of 35, 20, 28, 21, and 12.

4. Find the least common multiple of 19, 27, 18, 12, and 36. *Ans.* 2052.

5. Find the least common multiple of 2, 3, 4, 5, 11, 12, and 15.

6. Find the least common multiple of 21, 25, 12, 18, and 49. *Ans.* 44100.

7. Find the least common multiple of 9, 11, 14, 63, and 99. *Ans.* 1386.

8. Find the least common multiple of 10, 6, 42, 15, 30, 105, and 210.

27. THE GREATEST COMMON DIVISOR.

Any number that will exactly divide two or more other numbers, is called a *common divisor*, or *common measure*, and the greatest number that will so divide them, is the *greatest common divisor*, or *greatest common measure*, of those numbers. Thus, 2, 3, and 6, are all common divisors of 6, 12, 18, 24, and 30; but their greatest common divisor is 6. Any two numbers that have no common divisor, as 6 and 7, 11 and 15, 18 and 25, are said to be *prime to each other*.

EXAMPLE FOR THE BOARD.

The common divisor of any two numbers will also divide their difference. Thus 3 is contained in 27, 9 times, and in 36, 12 times. It must therefore be contained in 36 — 27, 12 times — 9 times, or 3 times.

```
384)672(1
    384
    ———
    288)384(1
        288
        ———
         96)288(3
            288
```

Let it then be required to find the greatest common divisor of 384 and 672. We first divide 672 by 384, to see if the smaller number will exactly divide the larger, and we find a remainder, 288. Now, the greatest common divisor of 384 and 672, is also the greatest common divisor of 288 and 384, because, if we suppose it contained 4 times in 384, and 7 times in 672, it must be contained 3 times in their difference, 288. Dividing 384 by 288, we find a remainder, 96. For the reason given above, the greatest common divisor sought, is also the greatest common divisor of 96 and 288. Dividing 288 by 96, we find it is contained exactly 3 times. 96 is therefore the greatest common divisor. Hence the following

RULE.

Divide all of the given numbers by the smallest number, and bring down the remainders. Divide the first divisor and all these remainders by the smallest remainder, and bring down as before. Continue this process, (at each time employing the LEAST *remainder as a new divisor), until you find a divisor that leaves no remainder. The divisor last used, will be the greatest common divisor.*

By resolving the several numbers into their prime factors, or by the table of prime factors, the greatest common divisor may sometimes be found more readily, in the following manner:

Form the product of the prime factors common to all the given numbers, employing each factor the least number of times it is used in either number.

EXAMPLE.

What is the greatest common divisor of 430, 602, 2150, and 3612?

By the table we find these numbers are equal, respectively, to 2 × 5 × 43, 2 × 7 × 43, 2 × 5 × 5 × 43, 2 × 2 × 3 × 7 × 43. The only factors common to all, are 2 × 43 = 86, which is the greatest common divisor.

28. EXAMPLES FOR THE PUPIL.

1. Find the greatest common divisor of 48, 72, and 60.

2. Find the greatest common divisor of 1001, 385, and 539. *Ans.* 77.

3. Find the greatest common divisor of 405, 567, 729, and 891.

4. Find the greatest common divisor of 2863 and 1151. *Ans.* 1.

5. Find the greatest common divisor of 992, 960, 928, and 32.

6. Find the greatest common divisor of 1177, 1391, and 1819. *Ans.* 107.

7. Find the greatest common divisor of 2943, 2616, and 4578. *Ans.* 327.

8. Find the greatest common divisor of 2148, 6444, and 3850.

VII. REVIEW.

29. Miscellaneous Examples.

1. How many days are there in a common year, there being 31 in January, 28 in February, 31 in March, 30 in April, 31 in May, 30 in June, 31 in July, 31 in August, 30 in September, 31 in October, 30 in November, and 31 in December?

2. Bought a piece of cloth for $137.50, and sold it again for $150. How much was gained by the sale?

3. Find the value of 325 barrels of flour, at $5 per barrel.

4. How many barrels of flour, at $6 per barrel, can be bought for $1986?

5. How many pounds are there in 1795 shillings, there being 20 shillings in a pound?

6. How many shillings in 13475 pence, there being 12 pence in a shilling? *Ans.* $1122\frac{11}{12}s.$

7. How many pence in 5775 farthings, a penny being equivalent to 4 farthings? *Ans.* 1443.75*d.*

8. How many strokes does a clock-hammer strike in a day? *Ans.* 156 *strokes.*

9. An orchard contains 14 rows of trees, and there are 37 trees in each row. How many trees are there in the whole orchard?

10. What would be the cost of a gallon of molasses, at $.0625 per pint? *Ans.* 50*cts.*

11. Two men are travelling in opposite directions, one at the rate of 59 miles a day, and the other at the rate of 63 miles in a day. How far apart will they be in a week? *Ans.* 854 *miles.*

12. A man was 51 years old after he had been married 28 years. At what age was he married?

13. A man was married at the age of 28. How old was he after he had been married 39 years?

14. A man was married at the age of 26. How long had he been married when he was 45 years old?

15. A piece of cloth which cost $137.50, was sold so as to gain $12.50. At what price was it sold?

16. If 17 men can do a piece of work in 59 days, in how many days could one man do it?

17. If 17 men can do a piece of work in 59 days, how many men could do it in 1 day?

18. If one man can do a piece of work in 1003 days, in how many days could 17 men do it?

19. If one man can do a piece of work in 1003 days, how many men could do it in 59 days?

20. A mile is 5280 feet. How many yards in 2 miles? *Ans.* 3520*yds.*

21. In what time would a cannon-ball fly 3 miles, moving 2000 feet per second? *Ans.* 7.92*sec.*

22. If a man's income is $2000 per annum, and his expenses are $3.25 per day, how much will he spend in a year? How much will he save in a year? 2*d Ans.* $813.75.

23. In what length of time will a second's pendulum tick 1000000 times? *Ans.* 11*dys.* 13*h.* 46*m.* 40 *sec.*

24. A gentleman received $418.50 from one man, $362.75 from another, and $1198.875 from a third. From the sum he paid $597.63. How much had he left? *Ans.* $1382.495.

25. Add 16.99, 259.087, 664, and from the sum subtract 774.876. *Ans.* 165.201.

26. A farmer who had 1984.65 bushels of wheat, sold 156.09 bushels to one man, 288.946 to another, and 476 to another. How much had he left? *Ans.* 1063.614*bu.*

27. 2976.3 and 484.95 and 2771, less 886.442, are how many?

28. A merchant deposits in a bank $1987.00 at one time, and $2411.75 at another. If he draws out $769.625, how much will remain in the bank?

29. Add 449, 164.4, 87, 6.91, and subtract 551.3 from their sum.

30. A flour dealer had 2864 barrels in store. How much had he left, after selling 95 barrels to one man, 160 to another, 479 to another, and 1192 to another?

31. 4954 and 495.4 and 49.54 and 4.954, less .4954, are how many?

32. Charles had 175 marbles. How many had he left, after giving 25 to William, 19 to George, 38 to Samuel, 49 to Peter, 41 to John, and losing 2?

33. 68 and 33.3 and 44.6 and 19.1 and 135, less 289, are how many? *Ans.* 11.

34. A vessel starting on a voyage of 2987 miles, sailed 729.4 miles the first week, 697.31 the second, and 753.08 the third. How far had she still to go? *Ans.* 807.21*m.*

35. Add 28.37, 56.091, 6.288, and 190, and from the sum subtract 213.75.

36. A tailor charged $29.75 for a coat, $48.375 for a cloak, and $5.875 for a vest. For some defect in the making, he deducted $9.28 from the bill. What remained to be paid?

37. Add 9988, 776.6, 55.44, 3.322, and .1991, and subtract from the sum 964. *Ans.* 9859.5611.

38. What is the difference between 9940897.62884 and 13988437.07?

39. If the smaller of two numbers is 9.87654321, and their difference is 4.0977387, what is the larger number?

40. If the larger of two numbers is 26.0084009, and their difference is 17.974298364, what is the smaller number?

41. A hogshead of sugar contained 587.965 pounds. How much remained in the hogshead, after 16.0084 pounds had been sold to one man, 96.944 pounds to another, and 205.87 pounds to another? *Ans.* 269.1426*lbs.*

42. 994876 and 994.876 and 6784.99 and 67849.9, subtracted from 2775083, leave how many?

Ans. 1704577.234.

43. A man bought five barrels of flour, at $4.375 per barrel, and sold it at $4.50 per barrel. How much did he make by the bargain?

44. 89320.014 and 17828 and 435162.5, less 115403.99 and 27916 and 599.0808, are how many?

Ans. 398391.4432.

45. A laborer receives $1.125 per day, and spends $0.64 per day for the support of his family. How much will he save in a week? (Remember that he *works* but six days.)

46. Subtract 99087.043899 from 127563.

47. What is .04 of 1; .04 of 2; .04 of 3; .04 of 4; .04 of 27?

48. What is .13 of 1; .13 of 2; .13 of 5; .13 of 13; .13 of 600?

49. What is .25 of 1; .25 of 7; .25 of 11; .25 of .25; .25 of 2500?

50. Per cent. signifies hundredths. What is 4 per cent. of 1; of 2; of 3; of 10; of 27?

51. What is 13 per cent. of 1; of 2; of 5; of 13; of 600?

52. What is 25 per cent. of 1; of 7; of 11; of 25; of 2500?

53. What is 6 per cent. of 27; 8 per cent. of 19; 13 per cent. of 80; 20 per cent. of .20?

54. If the rate of interest is 6 per cent. a year, in what time would it amount to 1 per cent.? What would be the rate for 4 months? For 6 months? For 10 months? For 18 months? For 2yr. 4mo.? For 3yr. 9mo.?

55. When the rate of interest is 6 per cent. a year, what is the rate for 6 days? For 12 days? For 18 days? For 42 days? For 25 days? For 39 days?

56. At 6 per cent. a year, what is the interest of $125 for 2½ years? For 3¾ years? For 5yr. 6mo.? For 2yr. 8mo.?

57. At 6 per cent. a year, what is the interest of $430 for 1yr. 4mo.? For 2yr. 2mo.? For 3yr. 10mo. 24dy.? For 1yr. 3mo. 18dy.?

58. At 6 per cent. per annum, what is the interest of $312.50 for 6mo. 24dy.? For 8mo. 14dy.? For 9mo. 12dy.? For 3yr. 3mo. 3dy.?

59. A broker sold goods for $2575. What was the amount of his commission, at 2 per cent.?

60. What sum must be paid to insure $1750 on a store, at 1 per cent.?

61. What will be the tax upon $5680, at .5 per cent. (.005)? *Ans.* $28.40.

62. What must be paid for $5000 worth of stock, at 94 per cent.?

63. A merchant sold goods which cost him $1700, so as to gain 10 per cent. What did he receive for the whole?

64. A merchant bought a cargo of flour for $2250, but in consequence of damage, he sold it so as to lose 6 per cent. What did he receive from the sale?

65. An invoice of merchandise was imported, valued at $837.50. What was the amount of the duties, at 15 per cent.?

66. Interest is computed at banks, where the rate is 6 per cent. per annum, by multiplying the principal by the

number of days, and dividing the product by 6000. What is the bank interest* of $1000 for 120 days? *Ans.* $20.

67. What is the bank interest of $275 for 90 days? For 63 days? For 33 days? For 123 days?

68. What is the bank interest of $394.75 for 37 days? For 45 days? For 82 days? For 10 days?

69. What is the bank interest of $1800 for 29 days? For 40 days? For 13 days?

70. What is the bank interest of $1220 for 11 days? For 28 days? For 31 days?

71. What is the difference between the bank interest and customary interest, on $1000 for 1 year (365 days), at 6 per cent.? *Ans.* $.83⅓.

72. A fodder of lead weighs 19 hundred-weight 2 quarters. To how many pounds is this equivalent, there being 28 pounds in a quarter, and 4 quarters in a hundred-weight?

73. Camillus, the Roman general, was fined 1500 libellæ, in consequence of the enmity of the Tribunes. Estimating the libella at 3 farthings, and the farthing at .25 of a cent, what was the value of the fine in our currency? *Ans.* $11.25.

74. A horse power is generally estimated as sufficient to raise 33000lb. 1 foot high in 1 minute. At this estimate, what must be the power of a machine which would raise 297000lb. 232 feet high in 4 minutes?

Ans. 522 *horse power.*

75. How much cloth would be required to furnish two suits to each scholar, in a charity-school of 83 pupils, if each suit should take 2 yards?

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* In all cases where the rate is not stated, it is supposed to be 6 per cent.

76. If two men set out at the same time from Boston and New York, respectively, and journey towards each other, one at the rate of 7 miles, and the other at the rate of 5 miles per hour, in what time will they meet, the distance of the two places apart being 216 miles?

77. If 9 men can reap 63 acres of wheat in 7 days, how much could they reap in 1 day? How much could 1 man reap in one day? 7 men in 1 day? 7 men in 9 days?

78. If 3 men can mow 24 acres of grass in 4 days, how much could 4 men mow in 3 days? 2 men in 6 days? 1 man in 12 days? 12 men in 1 day? 9 men in 9 days?

79. If 9 men can build 18 rods of wall in 2 days, how much could 2 men build in 9 days? 1 man in 18 days? 18 men in 1 day? 3 men in 6 days? 6 men in 3 days? 5 men in 4 days? 7 men in 8 days?

80. If 4 men in 5 days build 20 rods of wall, how many men could build 20 rods in 6 days? 15 rods in 3 days? 27 rods in 9 days? 48 rods in 12 days? 64 rods in 8 days? 132 rods in 11 days?

81. Find the least common multiple of 27, 16, 12, 18, 108, and 72. *Ans.* 432.

82. Find the greatest common divisor of 324, 972, 1188, 540, and 756. *Ans.* 108.

PRÓOF BY CASTING OUT NINES.

The figure 9 has the curious property of exactly dividing any number, when the sum of its digits is divisible by 9, and on this property is founded the mode of proof known as *casting out the nines.* This is done by adding the figures which compose any number, and rejecting 9 from the sum as often as possible. Thus, if we wish to cast out the 9's from 7683217, we say 7 and 6 are 13 *less* 9 *are* 4 and 8 are 12 *less* 9 *are* 3 and 3 are 6 and 2 are 8 and 1 are 9 *less* 9 *are* 0 and 7 are 7. We then know that if 7683217 is divided by 9, there will be a remainder, 7.

To prove any operation, we must reject the nines from each of the original numbers, perform the operation with the remainders, and reject the nines from the result, and also from the original result. If the work is right, the final remainders will be equal. The following examples will show the application of the rule:

Addition.	Rem.
168412	4
49037	5
684673	7
902122	16
Final rem. 7 =	7

Subtraction.	
23849047	1 + 9
9108736	7
14740311	3
Final rem. 3 =	3

Multiplication	Rem.
83869	7
233	8
251607	56
251607	
167738	
19541477	
Final rem. 2 =	2

Division.	
268)3462(12	
268	
782	
536	
246	
7 × 3 + 3 = 24	Rem. 6
Dividend rem.	6

In the example of subtraction, the remainder of the minuend being less than that of the subtrahend, we increase the upper number by one of the rejected nines.

The proof is not infallible, as, if any error is 9 or some multiple of 9, it will not be detected.

VIII. FRACTIONS.

30. If we divide 16 apples among 3 boys, we can give each of them 5, and have 1 left. If we wish to divide the remaining apple among them, we must cut it into 3 equal parts, and give one to each of them. Each of those parts would be called *one-third*, and written $\frac{1}{3}$.

Again, if we wished to divide 31 apples among 9 boys, we could give each of them 3, and have 4 left. To divide those 4, we might cut each one into 9 parts, or ninths, and give each boy 4 of the parts. Then $4 \div 9 = \frac{4}{9}$.

Numbers of this kind are called *Fractions*, or broken numbers, and may be regarded in three different lights. Thus, $\frac{3}{4}$ may be read *3-fourths*, $\frac{1}{4}$ *of* 3, or 3 *divided by* 4; $\frac{31}{56}$ is 31 *fifty-sixths*, $\frac{1}{56}$ of 31, or $31 \div 56$, &c.

The dividend, or *numerator*, may be considered as numbering the parts that are taken. The divisor, or *denominator*, marks the number of parts into which a unit is divided. The numerator and denominator are also called the *terms* of the fraction.

31. DEFINITIONS.

A *proper fraction*, is one in which the numerator is less than the denominator, and the fraction is therefore less than 1, as $\frac{3}{5}$, $\frac{7}{11}$, $\frac{22}{27}$.

An *improper fraction*, is one in which the numerator is equal to, or greater than the denominator. In the former case, the fraction is equal to 1,—as $\frac{11}{11}$, $\frac{3}{3}$, $\frac{19}{19}$. In the latter, it is more than 1, as $\frac{21}{7}$, which (as $\frac{7}{7} = 1$) is equal to 3; $\frac{19}{5} = 3\frac{4}{5}$; $\frac{17}{2} = 8\frac{1}{2}$. A whole number may always be regarded as the numerator of an improper fraction, whose denominator is 1. Thus, $12 = \frac{12}{1}$; $28 = \frac{28}{1}$.

A *mixed number*, consists of a whole number and a fraction, as $3\frac{2}{3}$, which (as $1 = \frac{3}{3}$) is equal to $\frac{11}{3}$; $126\frac{10}{13}$, which (as $1 = \frac{13}{13}$) is equal to $\frac{1648}{13}$.

A *compound fraction*, is a fraction of a fraction, as $\frac{1}{4}$ of $\frac{2}{3}$; $\frac{4}{7}$ of $\frac{3}{5}$ of $\frac{7}{11}$.

A *complex fraction*, is one which contains a fraction in its numerator or denominator, as $\frac{2\frac{1}{2}}{8}$; $\frac{1}{8\frac{1}{3}}$, $\frac{3\frac{1}{8}}{4\frac{2}{3}}$; $\frac{\frac{3}{4}}{\frac{7}{8}}$.

32. REDUCTION OF FRACTIONS.

An improper fraction may be reduced to a whole or mixed number, *by performing, or working out, the division which the fraction expresses.* Thus, $\frac{27}{11} = 27 \div 11 = 2\frac{5}{11}$. This reduction must always be made in giving the result of any operation.

A whole number may be reduced to a fraction having any given denominator, *by multiplying the whole number by the given denominator.* Thus $2 = \frac{22}{11} = \frac{26}{13} = \frac{8}{4}$.

A mixed number may be reduced to an improper fraction, *by reducing the whole number to a fraction, and adding the fraction.* Thus, since $2 = \frac{22}{11}$, $2\frac{5}{11} = \frac{22}{11} + \frac{5}{11} = \frac{27}{11}$.

A compound fraction may be reduced to a simple one, *by multiplying all the numerators together for a new numerator, and all the denominators for a new denominator.* Thus, $\frac{3}{4}$ of $\frac{5}{6} = \frac{15}{24}$. For $\frac{1}{4}$ of $\frac{1}{6} = \frac{1}{24}$, because, if we suppose a unit divided into 6 equal parts, and each part again divided into 4 equal parts, the whole unit will be divided into 24 equal parts, or 24ths. Then, $\frac{1}{4}$ of $\frac{5}{6} = \frac{5}{24}$, and $\frac{3}{4}$ of $\frac{5}{6} = \frac{15}{24}$, and so of any other similar fractions. *If the numerators and denominators have common factors, they may be rejected*, as in the following example:

Reduce $\frac{1}{2}$ of $\frac{2}{7}$ of $\frac{5}{9}$ of $\frac{14}{15}$ to a simple fraction.

The fraction, by the above rule, is equivalent to $\frac{1 \times 2 \times 5 \times 14}{2 \times 7 \times 9 \times 15}$.

Rejecting the common factors 2, 7, and 5, we have $\frac{1 \times 2}{9 \times 3} = \frac{2}{27}$.

The reason of this process will be evident if we remember that $\frac{2 \times 7 \times 5}{2 \times 7 \times 5} = 1$, and, dividing any number by 1, does not alter its value. Compound fractions must always be reduced to simple ones, before performing any operation.

33. EXAMPLES FOR THE BOARD.

1. Reduce $\frac{7965}{13}$ to a mixed number.

2. Reduce $\frac{4131}{27}$ to a whole number.

3. Reduce 95 to sevenths; to fifteenths; to forty-ninths.

4. Reduce $47\frac{3}{17}$ to an improper fraction.

5. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{14}{19}$ of $4\frac{2}{4}$ to a simple fraction, and explain the method of cancelling.

34. EXAMPLES FOR THE PUPIL.

1. Reduce to whole or mixed numbers, $\frac{149}{13}$; $\frac{270}{9}$; $\frac{759}{11}$; $\frac{421}{18}$.

2. Reduce 49 to 7ths; 12ths; 13ths; 60ths.

3. Reduce to an improper fraction, $4\frac{73}{75}$; $26\frac{1}{11}$; $149\frac{2}{3}$; $22\frac{7}{13}$.

4. Reduce to a simple fraction $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$; $\frac{2}{6}$ of $\frac{5}{9}$ of $\frac{12}{25}$ of $\frac{5}{8}$; $\frac{4}{7}$ of $\frac{14}{15}$ of $\frac{5}{8}$ of $\frac{2}{3}$; $\frac{3}{11}$ of $\frac{11}{14}$ of $\frac{7}{9}$ of $\frac{6}{18}$.

5. Reduce $19\frac{1}{19}$ to an improper fraction; $\frac{818}{32}$ to a mixed number; $\frac{2376}{27}$ to a whole number; 613 to 15ths; $\frac{2}{3}$ of $\frac{1}{6}$ of $\frac{18}{19}$ of $\frac{1}{2}$ to a simple fraction.

35. REDUCTION OF FRACTIONS.—CONTINUED.

A fraction may be reduced to a decimal, *by performing the division which the fraction expresses, annexing decimal zeroes to the numerator.* Thus, $\frac{3}{5} = 3.0 \div 5 = .6$.

A decimal may be reduced to a fraction, *by writing the decimal for a numerator, and the denomination tenth, hundredth, &c., for a denominator.* Thus, $.06 = \frac{6}{100}$; $.193 = \frac{193}{1000}$.

A fraction may be reduced to its lowest terms, *by dividing the numerator and denominator by their greatest common divisor* Thus, $\frac{65}{117} = \frac{5}{9}$, because $\frac{65}{117} \div \frac{13}{13} = \frac{5}{9}$, and $\frac{13}{13} = 1$. This reduction should always be made in giving the result of any operation.

If the denominator is 10, 100, 1000, &c., *Divide both terms of the fraction by* 10, 5, *and* 2, *as often as it can be done without a remainder.* This process will evidently effect the reductions desired, because 10 contains no prime factors other than 2 and 5. In the reduction of decimals, this rule will be found the most convenient.

36. EXAMPLES FOR THE BOARD.

1. Reduce to decimals $\frac{7}{32}$; $\frac{1}{16}$; $\frac{3}{7}$.

2. Reduce to a fractional form .093; .00941; 13.732.

3. Reduce to the lowest terms $\frac{44}{132}$; $\frac{87}{145}$; $\frac{693}{819}$.

37. EXAMPLES FOR THE PUPIL.

1. Reduce to a decimal, $\frac{7}{25}$; $\frac{3}{4}$; $\frac{1}{2}$; $\frac{8}{9}$; $\frac{7}{9}$; $\frac{6}{9}$; $\frac{5}{9}$; $\frac{4}{9}$; $\frac{3}{9}$; $\frac{2}{9}$; $\frac{1}{9}$; $\frac{5}{7}$.

2. Reduce to their lowest terms, $\frac{75}{90}$; $\frac{38}{76}$; $\frac{19}{57}$; $\frac{32}{44}$; $\frac{33}{44}$; $\frac{28}{91}$; $\frac{256}{304}$.

3. Reduce each of the following decimals to a fraction, and reduce the fraction to its lowest terms: .8; .014; .08; .008; .045; .1768; .0375; .25.

4. Reduce to their lowest terms, $\frac{364}{392}$; $\frac{2500}{10000}$; $\frac{765}{972}$; $\frac{345}{495}$; $\frac{3990}{5890}$.

5. Reduce to a decimal, $\frac{4}{7}$; $\frac{3}{7}$; $\frac{2}{7}$; $\frac{1}{7}$; $\frac{12}{14}$; $\frac{181}{273}$; $\frac{45}{773}$; $\frac{810}{1277}$; $\frac{463}{1804}$; $\frac{25}{3076}$.

6. Reduce $\frac{24 \times 21}{28 \times 36}$ to its lowest terms, by cancelling.

$$\frac{\overset{\cancel{4}}{\cancel{24}} \times \overset{\cancel{3}}{\cancel{21}}}{\underset{\cancel{4}}{\cancel{28}} \times \underset{\cancel{6}2}{\cancel{36}}} \qquad \textit{Ans. } \tfrac{1}{2}.$$

7. Reduce $\frac{12 \times 4 \times 5}{3 \times 2 \times 5}$ to its lowest terms, by cancelling.

8. Reduce $\frac{13 \times 8 \times 7}{7 \times 13 \times 2}$ to its lowest terms, by cancelling.

9. Reduce $\frac{6 \times 5 \times 4}{9 \times 10 \times 12}$ to its lowest terms, by cancelling.

10. Reduce $\frac{15 \times 18}{30 \times 9}$ to its lowest terms, by cancelling.

11. Divide the product of 40 × 42 by the product of 21 × 16, by cancelling.

12. Divide the product of 45 × 54 by the product of 18 × 27, by cancelling.

13. Divide the product of 75 × 24 by the product of 60 × 15, by cancelling.

14. Divide the product of 72 × 18 by the product of 48 × 27, by cancelling.

15. Reduce the following fractions to their lowest terms, by cancelling: $\frac{192}{320}$; $\frac{196}{420}$; $\frac{552}{736}$; $\frac{231}{1001}$.

38. REDUCTION OF FRACTIONS.—CONTINUED.

Two or more fractions may be reduced to a common denominator, *by dividing the least common multiple of all the denominators, by each given denominator, and multiplying both terms of the fraction by the quotient.*

EXAMPLE FOR THE BOARD.

If we wish to reduce the fractions, $\frac{5}{9}$, $\frac{2}{3}$, $\frac{7}{12}$, $\frac{13}{18}$, and $\frac{5}{6}$ to a common denominator, we first find the least common multiple, which is 36. Then,

$$36 \div 9 = 4 \text{ and } \tfrac{5}{9} \times \tfrac{4}{4} = \tfrac{20}{36}$$
$$36 \div 3 = 12 \text{ and } \tfrac{2}{3} \times \tfrac{12}{12} = \tfrac{24}{36}$$
$$36 \div 12 = 3 \text{ and } \tfrac{7}{12} \times \tfrac{3}{3} = \tfrac{21}{36}$$
$$36 \div 18 = 2 \text{ and } \tfrac{13}{18} \times \tfrac{2}{2} = \tfrac{26}{36}$$
$$36 \div 6 = 6 \text{ and } \tfrac{5}{6} \times \tfrac{6}{6} = \tfrac{30}{36}$$

or, as there are $\tfrac{36}{36}$ in 1,

$$\tfrac{1}{9} = \tfrac{4}{36} \text{ and } \tfrac{5}{9} = \tfrac{20}{36}$$
$$\tfrac{1}{3} = \tfrac{12}{36} \text{ and } \tfrac{2}{3} = \tfrac{24}{36}$$
$$\tfrac{1}{12} = \tfrac{3}{36} \text{ and } \tfrac{7}{12} = \tfrac{21}{36}$$
$$\tfrac{1}{18} = \tfrac{2}{36} \text{ and } \tfrac{13}{18} = \tfrac{26}{36}$$
$$\tfrac{1}{6} = \tfrac{6}{36} \text{ and } \tfrac{5}{6} = \tfrac{30}{36}$$

In the first mode of performing the operation, the multipliers are each equal to 1, and the product by 1 is of course equal to the multiplicand.

39. EXAMPLES FOR THE PUPIL.

1. Reduce to a common denominator, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{5}{12}$, $\frac{1}{2}$, and $\frac{1}{4}$.

2. Reduce to a common denominator, $\frac{6}{7}$, $\frac{3}{8}$, $\frac{5}{14}$, and $\frac{1}{4}$.

3. Reduce to a common denominator, $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{5}$, $\frac{2}{3}$, $\frac{1}{2}$ of $\frac{3}{5}$, and $\frac{1}{3}$ of $\frac{2}{5}$ of 3.

The compound fractions must first be reduced to simple ones.

4. Reduce to a common denominator, $\frac{4}{5}$ of $\frac{2}{9}$, $\frac{7}{15}$, $\frac{2}{3}$ of $\frac{1}{8}$, $\frac{11}{40}$, and $\frac{1}{4}$ of $\frac{2}{3}$.

5. Reduce $29\frac{163}{164}$ to an improper fraction; $\frac{9847}{131}$ to a mixed number; $\frac{287}{1463}$ to a decimal; $\frac{793}{1037}$ to its lowest terms; $\frac{19}{36}$, $\frac{7}{54}$, and $\frac{4}{9}$ to a common denominator; .004625 to a fraction; 295 to 19ths.

40. ADDITION OF FRACTIONS.

The addition of fractions having the same denominator, is as easy as the addition of any other numbers. Thus, as 5 houses + 3 houses + 2 houses = 10 houses, so $\frac{5}{7} + \frac{3}{7} + \frac{2}{7} = \frac{10}{7}$, and $\frac{5}{13} + \frac{3}{13} + \frac{2}{13} = \frac{10}{13}$. Therefore, to add fractions, *Reduce all the given fractions to a common denominator, and add their numerators.*

41. EXAMPLES FOR THE BOARD.

1. Add $4\frac{7}{9}$, $13\frac{3}{4}$, $\frac{1}{2}$ of $8\frac{5}{6}$, and $\frac{11}{12}$.
2. Add $\frac{5}{8}$, $\frac{13}{20}$, 2.7, 5.75, and $\frac{2}{7}$ of $8\frac{2}{5}$.

42. EXAMPLES FOR THE PUPIL.

1. Add $2\frac{1}{3}$, $\frac{1}{2}$, $15\frac{1}{4}$, and $28\frac{5}{6}$.
2. Add $\frac{2}{3}$ of $\frac{1}{4}$, $\frac{1}{2}$ of $\frac{5}{6}$, and $19\frac{3}{4}$. *Ans.* $20\frac{1}{3}$.
3. Add $\frac{5}{9}$, $\frac{3}{7}$, $\frac{11}{21}$, and $\frac{19}{63}$.
4. Add 19, $28\frac{1}{2}$, $16\frac{1}{11}$, and $3\frac{9}{14}$. *Ans.* $67\frac{18}{77}$.
5. Add $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{5}$ of $\frac{6}{7}$, $2\frac{7}{10}$, $\frac{10}{21}$, and $15\frac{19}{35}$. *Ans.* $18\frac{157}{210}$.
6. Add $12\frac{3}{4}$, $\frac{6}{7}$ of $\frac{5}{8}$, $11\frac{5}{9}$, $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$, and $19\frac{1}{6}$.
7. Add $\frac{1}{3}$ of $\frac{2}{5}$ of $\frac{4}{7}$ of 9, $\frac{1}{11}$, $\frac{5}{7}$ of $\frac{2}{3}$, $\frac{14}{15}$ and $7\frac{19}{35}$. *Ans.* $9\frac{842}{1155}$.

43. SUBTRACTION OF FRACTIONS.

As 8 houses — 3 houses = 5 houses, so $\frac{8}{9} - \frac{3}{9} = \frac{5}{9}$, and $\frac{8}{14} - \frac{3}{14} = \frac{5}{14}$. The subtraction of fractions having a common denominator, is, therefore, devoid of difficulty, except when the numerator of the subtrahend is greater than that of the minuend.

44. EXAMPLE FOR THE BOARD.

$$\begin{array}{r} 16\frac{2}{9} \\ 8\frac{5}{9} \\ \hline \end{array} \qquad \begin{array}{r} 16\frac{11}{9} \\ 4\frac{5}{9} \\ \hline 12\frac{6}{9} \end{array}$$

If we wish to subtract $8\frac{5}{9}$ from $16\frac{2}{9}$, we must, as in simple subtraction, increase the fraction in the minuend by 1, or $\frac{9}{9}$, and carry 1 to the units of the subtrahend. Therefore, to subtract fractions,

Reduce both fractions to a common denominator, and subtract the numerator of the subtrahend from the numerator of the minuend. If the numerator of the minuend is *less than that of the subtrahend, increase it by the denominator, and carry* 1 *to the units figure of the subtrahend.*

45. EXAMPLES FOR THE PUPIL.

1. What is the difference between $\frac{5}{9}$ and $\frac{6}{11}$?
2. Subtract $3\frac{1}{5}$ from $11\frac{7}{8}$.
3. $1\frac{4}{7} - \frac{12}{13} = ?$ *Ans.* $\frac{59}{91}$.
4. Subtract $49\frac{5}{243}$ from $56\frac{1}{243}$. *Ans.* $6\frac{239}{243}$.
5. From $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $7\frac{1}{8}$ take $1\frac{37}{43}$. *Ans.* $\frac{2015}{5504}$.
6. What is the difference between $4\frac{19}{87}$ and $2\frac{3}{4}$?
7. What is the difference between $\frac{1}{3}$ of $\frac{10}{11}$ and $\frac{2}{5}$ of $\frac{4}{9}$?
8. Subtract $\frac{4}{5}$ of $16\frac{1}{8}$ from $\frac{3}{8}$ of $\frac{1}{2}$ of $95\frac{2}{3}$.
Ans. $4\frac{209}{240}$.

46. MULTIPLICATION OF FRACTIONS.

If any number be multiplied by $\frac{3}{4}$, it is evident that the product will be $\frac{3}{4}$ as large as if multiplied by 1. Then, $\frac{3}{4} \times 11 = \frac{3}{4}$ of $\frac{11}{1}$; $\frac{2}{7} \times \frac{5}{8} = \frac{2}{7}$ of $\frac{5}{8}$. Multiplication of fractions is therefore performed in the same way as reduction of compound fractions to simple ones. Then, when either the multiplier or multiplicand is a fraction, *Change whole or mixed numbers, (if any,) to improper fractions, and multiply the numerators together for a new numerator, and the denominators for a new denominator. Cancel, if possible, before multiplying.*

When the multiplier is a mixed number, the product may often be obtained most conveniently, *by multiplying first by the whole number, and afterwards by the fraction, and adding the two products.*

47. EXAMPLES FOR THE BOARD.

1. Multiply $\frac{3}{7}$ of $1\frac{5}{9}$ by $28\frac{1}{2}$.
2. Multiply $4\frac{6}{13}$ by $3\frac{17}{29}$.
3. Multiply $\frac{2}{3}$ of $\frac{6}{11}$ of $12\frac{4}{7}$ by $\frac{7}{8}$ of $\frac{4}{13}$ of $3\frac{7}{15}$.

48. EXAMPLES FOR THE PUPIL.

1. Multiply $\frac{14}{15}$ by $3\frac{1}{9}$; $2\frac{7}{8}$ by $5\frac{1}{3}$. *Ans.* $2\frac{122}{135}$; $15\frac{1}{3}$.
2. Multiply $\frac{2}{7}$ of $\frac{8}{11}$ by $16\frac{4}{15}$. *Ans.* $3\frac{439}{1155}$.
3. Multiply $15\frac{7}{8}$ by $9\frac{1}{4}$; $28\frac{11}{13}$ by $1\frac{6}{17}$. *Ans.* $146\frac{27}{32}$; $39\frac{6}{221}$.
4. Multiply $\frac{341}{96}$ by 4; by 8; 12; 16; 24; 96.
5. Multiply $22\frac{3}{5}$ by $\frac{1}{2}$ of $\frac{4}{5}$ of $16\frac{11}{12}$. *Ans.* $152\frac{139}{150}$.
6. $14\frac{3}{19} \times 15\frac{4}{5} = ?$ $8\frac{6}{7} \times 41\frac{8}{9} = ?$ *Ans.* $223\frac{66}{95}$; $371\frac{1}{63}$.
7. Multiply $\frac{191}{248}$ by 2; 4; 31; 8; 248.
8. Multiply $\frac{2}{13}$ of $\frac{9}{11}$ by $\frac{4}{5}$ of $\frac{3}{7}$ of $9\frac{1}{15}$. *Ans.* $\frac{9792}{25025}$.
9. What is the product of $33\frac{1}{3}$ by $4\frac{1}{2}$? by $1\frac{1}{2}$? by $15\frac{3}{4}$?

49. DIVISION OF FRACTIONS.

The smaller the divisor, the more times will it be contained in the dividend, and the larger will be the quotient. Thus, if $\frac{2}{3}$ be divided by 1, the quotient will be $\frac{2}{3}$. $\frac{1}{4}$ will be contained 4 times as often as 1; therefore, $\frac{2}{3} \div \frac{1}{4} = \frac{4}{1} \times \frac{2}{3}$. $\frac{3}{4}$ will be contained only $\frac{1}{3}$ as often as $\frac{1}{4}$; therefore, $\frac{2}{3} \div \frac{3}{4} = \frac{1}{3}$ of $\frac{4}{1} \times \frac{2}{3} = \frac{4}{3} \times \frac{2}{3}$. Then, whenever either the divisor or dividend is a fraction, *Change whole or mixed numbers, (if any,) to improper fractions, and, if the numerators and denominators cannot be directly divided into each other, invert the divisor, and proceed as in multiplication.*

COMPLEX FRACTIONS, or such as contain fractions in their numerator or denominator, or both, may be resolved into simple fractions by this rule. Thus, $\frac{3\frac{5}{7}}{\frac{4}{5}} = 3\frac{5}{7} \div \frac{4}{5} = \frac{26}{7} \times \frac{5}{4} = 4\frac{9}{14}$.

50. EXAMPLES FOR THE BOARD.

1. Divide $4\frac{11}{13}$ by $\frac{27}{104}$.
2. Divide $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{14}{25}$ by $\frac{5}{9}$ of $\frac{8}{10}$ of $1\frac{7}{8}$.
3. Reduce to simple fractions, $\frac{2\frac{5}{9}}{10}$; $\frac{1}{8\frac{2}{3}}$; $\frac{\frac{6}{8}}{13}$.

51. EXAMPLES FOR THE PUPIL.

1. Divide $\frac{12}{35}$ by $\frac{6}{7}$; by $\frac{3}{5}$; by $\frac{4}{35}$.
2. Divide $2\frac{11}{13}$ by $5\frac{1}{2}$; by 14; by $8\frac{2}{7}$. *Ans.* $\frac{74}{143}$; $\frac{37}{182}$; $\frac{259}{754}$.
3. Divide $\frac{1}{2}$ of $\frac{5}{8}$ of $\frac{8}{7}$ by $6\frac{9}{11}$; by 10.
4. Divide 4 by $11\frac{3}{16}$; by $81\frac{4}{7}$. *Ans.* $\frac{64}{179}$; $\frac{28}{571}$.
5. What is the quotient of $7\frac{8}{9}$ by $10\frac{1}{12}$? By 15?
6. $\frac{2}{7}$ of $\frac{2}{3}$ of $\frac{1}{8}$ of $4 \div \frac{1}{6}$ of $\frac{5}{9}$ of $13\frac{1}{2} = ?$ *Ans.* $\frac{8}{105}$.
7. Divide $\frac{91}{126}$ by $4\frac{1}{3}$; by $\frac{7}{18}$; by $3\frac{1}{2}$.
8. Divide $\frac{7}{16}$ of $19\frac{2}{7}$ by $\frac{9}{10}$ of $\frac{3}{4}$ of $8\frac{1}{2}$. *Ans.* $1\frac{8}{17}$.
9. Divide $4\frac{12}{23}$ by $8\frac{14}{19}$; by $16\frac{1}{2}$; by $9\frac{5}{6}$. *Ans.* $\frac{988}{1909}$; $\frac{208}{759}$; $\frac{624}{1357}$.

IX. CIRCULATING, OR INFINITE DECIMALS.

52. IN changing fractions to decimals, if the divisor should contain any prime factors, other than 2 or 5, we shall always find that our work would continue without end;—the same figures being repeated again and again. The decimal is then called a circulating, or infinite decimal. Thus, $\frac{1}{3} = .333 +$; $\frac{2}{9} = .22222 +$; $\frac{1}{11} = .090909 +$; $\frac{317}{444} = .71396396 +$. The figures that are thus repeated, are called the *repetend.* The figures preceding the repetend, (if any,) are called the *finite part* of the decimal. The repetend is usually distinguished by placing a point over the first and last figure. Thus, $.\dot{3}$ is the same as $.3333 +$; $.\dot{0}\dot{9}$ is the same as $.090909 +$; $.71\dot{3}9\dot{6}$ is the same as $.71396396 +$. In the latter example, .71 is the *finite part,* and 396 is the repetend.

We find, by reducing the following fractions to decimals, that $\frac{1}{9} = .\dot{1}$; therefore, $\frac{2}{9} = .\dot{2}$, $\frac{5}{9} = .\dot{5}$, &c. $\frac{1}{99} = .\dot{0}\dot{1}$; therefore, $\frac{2}{99} = .\dot{0}\dot{2}$, $\frac{14}{19} = .\dot{1}\dot{4}$, $\frac{95}{99} = .\dot{9}\dot{5}$, &c. $\frac{1}{999} = .\dot{0}0\dot{1}$; therefore, $\frac{2}{999} = .\dot{0}0\dot{2}$, $\frac{45}{999} = .\dot{0}4\dot{5}$, $\frac{163}{999} = .\dot{1}6\dot{3}$, &c.

Hence, *every repetend is equivalent to a fraction, having the repetend for its numerator, and an equal number of* 9's *for its denominator.*

53. REDUCTION OF INFINITE DECIMALS.

TO REDUCE INFINITE DECIMALS TO FRACTIONS.—We have seen that $71.\dot{3}4\dot{2} = 71\frac{342}{999} = \frac{71271}{999}$. Therefore, $.071\dot{3}4\dot{2}$ (which is $\frac{1}{1000}$ of $71.\dot{3}4\dot{2}$), $= \frac{71271}{999000}$. In this example, the numerator is equivalent to $71342 - 71$; that is, to the whole decimal, *minus* its finite part. The denominator contains as many 9's as there are figures in the repetend, with as many 0's as there are finite figures. Hence we obtain the following

RULE.

If the repetend commences at tenths, consider the repetend as the numerator, and an equal number of 9's *as the denominator. If there are integral figures in the repetend, annex as many zeroes to the numerator. If the decimal has* *a finite part, then from the decimal, considered as a whole number, subtract the finite part for a numerator. For a denominator, write as many 9's as there are figures in the repetend, annexing as many zeroes as there are finite figures.*

54. EXAMPLES FOR THE PUPIL.

1. Reduce $.\dot{0}1\dot{8}$ to a fraction, and reduce the fraction to its lowest terms. *Ans.* $\frac{2}{111}$.

2. Reduce each of the following numbers to a fraction: $\dot{2}.\dot{7}$; $\dot{8}9.\dot{1}$; $.\dot{0}0\dot{4}$; $.\dot{0}51\dot{3}$; $\dot{6}.64\dot{9}$.

Ans. $\frac{30}{11}$; $\frac{3300}{37}$; $\frac{4}{999}$; $\frac{57}{1111}$; $\frac{66490}{9999}$.

3. Reduce each of the following numbers to a fraction: $.16\dot{3}9\dot{4}$; $.852\dot{9}0\dot{6}$; $\dot{2}45.\dot{9}$; $.06\dot{8}4\dot{3}$.

55. EXAMPLES FOR THE BOARD.

In ADDITION and SUBTRACTION of circulating decimals, if we wish to obtain the exact repetend, it will be necessary to *change all the given repetends into others, containing as many figures as the least common multiple of the number of places in each repetend.*

1. Add 17.5, $3.\dot{7}$, $419.0\dot{8}7\dot{5}$, $1.985\dot{6}\dot{3}$, and $32.\dot{1}27\dot{8}$.

The numbers of repetend figures are 0, 1, 3, 2, and 4; the least common multiple of which is 12. The common repetend must therefore consist of 12 figures, commencing at the lowest place of the given repetends, which is *ten-thousandths.* The numbers will be written as follows:

$$\begin{array}{r}
17.500\dot{0}0000000000\dot{0}\\
3.777\dot{7}7777777777\dot{7}\\
419.087\dot{5}8758758758\dot{7}\\
1.985\dot{6}3636363636\dot{3}\\
32.127\dot{8}1278127812\dot{7}\\
\hline
474.478\dot{8}1451027985\dot{6}
\end{array}$$

Adding the repetends, we find their sum is $\frac{2814510279864}{999999999999}$. Dividing by the rule for division by nines, we obtain a quotient $2\dot{8}1451027985\dot{6}$. The repetend is written down, and the 2 carried to the column of thousandths. Repetends, that thus commence and end at the same decimal places, are called *similar and conterminous.*

2. Subtract $1.7\dot{4}\dot{2}$ from $2.\dot{9}8\dot{7}$.

$$\begin{array}{r} 2.9\dot{8}79879\dot{9} \\ 1.7\dot{4}24242\dot{2} \\ \hline 1.1\dot{9}5518\dot{6} \end{array}$$

The common repetends have 6 figures. From the right-hand figure of the remainder we subtract 1, because the repetend of the minuend is less than that of the subtrahend. The reason of this subtraction will become evident, if we change the repetends to fractions, and subtract $1.7\frac{424242}{999999}$ from $2.9\frac{879879}{999999}$.

56. EXAMPLES FOR THE PUPIL.

1. Add $17.6\dot{9}$, $18\dot{3}$, 25.75, $3.2\dot{7}\dot{6}$, $194.\dot{4}\dot{3}$, and 649.287.

2. Add $3.2\dot{1}\dot{9}$, $63.\dot{3}7\dot{4}$, $285.\dot{1}\dot{2}$, 38.4, $.0\dot{3}7\dot{1}$, and $43.6\dot{8}$.
Ans. $433.8\dot{4}0804\dot{4}$.

3. Subtract $49.\dot{2}87\dot{1}$ from 64.

4. Subtract $215.9\dot{9}3\dot{1}$ from $1842.\dot{2}43\dot{4}$.
Ans. $1626.2\dot{5}0231149241\dot{0}$.

5. Subtract 11.27 from $30.\dot{4}0\dot{9}$. *Ans.* $19.13\dot{9}4\dot{0}$.

6. Subtract $2856.\dot{0}3\dot{6}$ from $3017.\dot{6}259\dot{1}$.
Ans. $161.\dot{5}8988022812655\dot{5}$.

7. Subtract $43.76\dot{3}$ from $288.1\dot{9}5\dot{4}$.

8. Add $21.\dot{3}$, 28.72, $6.\dot{4}\dot{7}$, $19.3\dot{4}\dot{5}$, $201.1\dot{5}9\dot{3}$, and $419.\dot{6}6243\dot{4}$. *Ans.* $696.69\dot{5}3293\dot{7}$.

9. Add 7.83, $24.\dot{1}$, $79.1\dot{4}\dot{2}$, $252.\dot{4}16\dot{3}$, and $17.3\dot{0}8\dot{7}$.
Ans. $380.80\dot{8}5856964074\dot{7}$.

10. Subtract $4.1\dot{9}5\dot{6}$ from $21.28\dot{4}3911\dot{3}$.
Ans. $17.08\dot{8}6954\dot{3}$.

57. EXAMPLE FOR THE BOARD.

In DIVISION OF CIRCULATING DECIMALS, we may adopt the following rule:

Make the repetends of the divisor and dividend similar and conterminous, and from the result, considered as whole numbers, subtract the finite part of each. Perform the division with the remainders as with whole numbers, and the true quotient will be obtained.

Divide $36.\dot{9}\dot{1}$ by $5.\dot{2}7\dot{3}$.

$$\begin{array}{r} 5.\dot{2}7327\dot{3})36.\dot{9}1919\dot{1} \\ 5 \qquad\qquad 36 \\ \hline 5273268)36919155(7 \\ 36912876 \\ \hline 6279 \end{array}$$

The exact fractional quotient is $7\frac{6279}{5273268}$, or $7\frac{23}{19316}$. The effect of subtracting the finite parts of the divisor and dividend, is the same as reducing the two numbers to improper fractions, and dividing the numerators.

58. EXAMPLES FOR THE PUPIL.

1. Divide $27.\dot{5}$ by $3.\dot{3}4\dot{1}$. *Ans.* 8.24667.
2. Divide $8.9\dot{7}$ by $42.\dot{8}1\dot{5}$. *Ans.* .209683.
3. Divide 39 by $63.\dot{4}32\dot{8}$. *Ans.* .614823.
4. Divide $4.0\dot{2}\dot{1}$ by $65.\dot{3}7\dot{8}$. *Ans.* .061506.
5. Divide $17.5\dot{8}32\dot{4}$ by 29.8. *Ans.* .590042.
6. Divide $41.31\dot{9}$ by $28.\dot{7}235\dot{8}$. *Ans.* 1.438539.
7. Divide $121.6\dot{2}\dot{3}$ by $24.\dot{0}18\dot{4}$. *Ans.* 5.063752.
8. Divide $75.8\dot{1}\dot{4}$ by $364.\dot{2}\dot{5}$. *Ans.* .208136.
9. Divide $4.30\dot{9}\dot{7}$ by $18.6\dot{1}584\dot{3}$.
10. Divide $768.\dot{4}3\dot{2}$ by $29.5\dot{7}96\dot{1}$.
11. Divide $44.\dot{3}808\dot{1}$ by 27.85.

X. COMPOUND NUMBERS.

59. WHEN several denominations of the same kind, (as pounds, shillings, and pence; miles, feet, and inches; gallons, quarts, and pints,) are embraced in one sum, they are called COMPOUND NUMBERS. The operations upon them are performed by regarding each denomination as a fraction of the next higher.

TABLES OF MEASURES, WEIGHTS, AND CURRENCIES.

I. MEASURES.

1. LONG MEASURE.

12	inches (in.)	make 1	foot.	ft.
3	feet	make 1	yard.	yd.
5½	yards	make 1	rod or pole.	r.
40	rods	make 1	furlong.	fur.
8	furlongs	make 1	mile.	m.
3	miles	make 1	league.	lea.
69⅙	miles, or geographical miles,	make 1	degree.	°.
60	geographical miles,			

360 degrees make the earth's circumference, 24876 m.

A palm is 3 inches; a hand is 4 inches; a span is 9 inches; a pace is 3 feet; a fathom is 6 feet; a knot is a geographical mile.

The English standard unit of Long Measure is the yard, which is equivalent to $\frac{360000}{391393}$ of the length of a "pendulum vibrating seconds of mean time in the latitude of London, in a vacuum at the level of the sea."* The United States standard, the original, of which the State standards are copies, is a brass scale of 82 inches in length, prepared for the survey of the coast of the United States, by Troughton, of London, and deposited in the Office of Weights and Measures, at Washington.

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* McCulloch.

The Chinese li	is	682	yards.
The Russian werst		1167	"
The ancient Roman mile		1614	"
The Tuscan mile		1808	"
The Turkish berri		1826	"
The Arabian mile		2148	"
The Irish mile		2240	"
The Polish mile		6075	"
The Persian parasang		6086	"
The German short mile		6859	"
The Dutch mile		8100	"
The Prussian mile		8237	"
The Danish mile		8244	"
The Dantzic mile		8475	"
The Hungarian mile		9113	"
The Swiss mile		9153	"
The German long mile		10126	"
The Hanover mile		11559	"
The Swedish mile		11700	"

2. CHAIN MEASURE.

7.92 inches	make 1 link.	l.
25 links	make 1 pole.	p.
4 poles	make 1 chain.	ch.
10 chains	make 1 furlong.	fur.
8 furlongs	make 1 mile.	m.

A lot of land measuring 10 chains in length and 1 in breadth, is an acre.

3. DUODECIMAL MEASURE.

12 fourths (′′′′)	make 1 third.	′′′
12 thirds	make 1 second.	′′
12 seconds	make 1 prime.	′
12 primes	make 1 unit.	

In the result of any operation, the unit is considered as a linear, a square, or a cubic foot; but, in multiplication, it must always be remembered that THE MULTIPLIER IS AN ABSTRACT NUMBER.

4. ASTRONOMICAL, OR CIRCULAR MEASURE.

60 seconds (″)	make 1 minute.	′
60 minutes	make 1 degree.	°
30 degrees	make 1 sign of the zodiac.	s
12 signs	make the orbit of a planet.	

The circumference of *every circle* is divided into 360 degrees, and the degrees are subdivided into minutes and seconds.

5. CLOTH MEASURE.

$2\frac{1}{4}$ inches	make 1 nail.	na.
4 nails	make 1 quarter.	qr.
4 quarters	make 1 yard.	yd.

39.371 inches make 1 French metre.
47.245 inches make 1 French aune, or ell.
22.98 inches make 1 braccio of Leghorn.
24.8 inches make 1 silk braccio of Venice.
26.46 inches make 1 woollen braccio of Venice.
28 inches make 1 arsheen, or Russian ell.
22.25 inches make 1 Leipsic ell.
27.36 inches make 1 Brabant ell.

6. FRENCH LONG MEASURE.

10 millimetres	make 1 centimetre.
10 centimetres	make 1 decimetre.
10 decimetres	make 1 METRE.
10 metres	make 1 decametre.
10 decametres	make 1 hectometre.
10 hectometres	make 1 kilometre.
10 kilometres	make 1 myriametre.

1 Metre = 39.371 English inches.

The *toise usuelle* = 2 metres. The *pied* or foot is $\frac{1}{3}$ of a metre. The *aune* or ell = $1\frac{1}{5}$ metres, or $47\frac{1}{4}$ English inches.

The Metre is one ten-millionth part of the distance from the pole to the equator.

7. SQUARE MEASURE.

144	square inches	make 1 square foot.
9	square feet	make 1 square yard.
30¼	square yards	make 1 square rod.
40	square rods	make 1 rood.
4	roods, or	make 1 acre.
10	square chains	
640	acres	make 1 square mile.

8. FRENCH SQUARE MEASURE.

The unit is the ARE, which is a square decametre, equivalent to 3.9539 square rods. The decimal multiples and submultiples, are the same as in the other tables.

9. CUBIC MEASURE.

1728 cubic inches	make 1 cubic foot.
50 feet of round timber	make 1 ton.
40 feet of hewn timber	
16 cubic feet	make 1 cord foot.
8 cord feet	make 1 cord.
42 cubic feet	make 1 ton of shipping.

The French unit of cubic measure, is the STERE, which is a cubic metre, with the decimal multiples and submultiples. The Stere is equivalent to 35.3171 cubic feet.

10. COMMON LIQUID MEASURE.

4 gills (gi.)	make 1 pint.	pt.
2 pints	make 1 quart.	qt.
4 quarts	make 1 gallon.	gal.
63 gallons	make 1 hogshead.	hhd.

The United States standard for measuring liquids, is the gallon, which is a vessel containing 58372.2 grains, (8.3389 pounds avoirdupois,) of the standard pound of distilled water, at the temperature of 39°.83 Fahrenheit, the vessel being weighed in air in which the barometer is 30 inches at 62° Fahrenheit. This corresponds very nearly with the English wine gallon, which contains 231 cubic inches. Milk and malt liquors are sold by Beer Measure in many places, the beer gallon containing 282 cubic inches. The hogshead (measure) is used only in estimating the contents of cisterns, wells, or large bodies of water.

11. APOTHECARIES' MEASURE.

60 minims (♏)	make 1 fluidrachm.	fʒ.
8 fluidrachms	make 1 fluidounce.	f℥.
16 fluidounces	make 1 pint.	O.
8 pints	make 1 gallon.	Cong.

A drop of water is *about* equal to 1½ minims, or 45 drops to a fluidrachm. A teacup holds *about* 4 fluidounces; a wineglass, 2 fluidounces; a tablespoon, ½ fluidounce; a teaspoon, 1 fluidrachm.

A pint of distilled water weighs 1.04139 lb. avoirdupois.

12. DRY MEASURE.

2 pints (pt.)	make 1 quart.	qt.
8 quarts	make 1 peck.	pk.
4 pecks	make 1 bushel.	bu.
36 bushels of coal	make 1 chaldron.	chal.

The United States standard is the bushel measure, containing 543391.89 standard grains, (77.6274 pounds Avoirdupois,) of distilled water, at the temperature of 39°.83 Fahrenheit, and barometer 30 inches at 62° Fahrenheit. This corresponds very nearly with the Winchester bushel, which is a cylinder 18½ inches in diameter, and 8 inches deep, containing 2150.42 cubic inches.

The French unit of capacity is the LITRE, which is a cubic decimetre, equivalent to 1.0517 wine quarts. The boisseau usuel = 12.5 litres. The litron usuel is the same as the litre, and is equivalent to 1.074 Paris pintes.

13. Imperial Liquid and Dry Measure.*

4 gills	make 1 pint.
2 pints	make 1 quart.
2 quarts	make 1 pottle.
2 pottles	make 1 gallon.
2 gallons	make 1 peck.
4 pecks	make 1 bushel.
4 bushels	make 1 coom.
2 cooms	make 1 quarter.

The gallon "contains 10 lb. avoirdupois weight of distilled water, weighed in air at the temperature of 62° of Fahrenheit's thermometer, the barometer being at 30 inches." This is equivalent to 277.274 cubic inches.

14. Time Measure.

60 seconds (sec.)	make 1 minute.	min.
60 minutes	make 1 hour.	h.
24 hours	make 1 day.	dy.
7 days	make 1 week.	
30 days	make 1 month.	mo.
365 days	make 1 common year.	yr.
366 days	make 1 leap year.	

365 days 5 hours 48 min. 50 sec. make 1 Solar Year.

365 days 6 hours make 1 Julian Year.

The leap years are those that are divisible by 4, as 1844, 1848; except the centurial years, which are leap years when the number of hundreds is divisible by 4, as 1600, 2000, &c.

EXAMPLES FOR THE PUPIL.

1. How many feet in 24 inches? In 36 inches? In 72 inches? In 144 inches? In 1728 inches? In 10 yards? In 13 yards? In 145 yards?

2. How many inches in 2ft.? In 7ft.? In 1yd.? In 9yd.? In 1 rod? In 5r.? In 1 furlong?

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* This is the standard measure used in Great Britain.

3. How many furlongs in 80 rods? In 160r.? In 520r.? In 2 miles? In 17 miles? In 3 leagues?

4. How many minutes in 120 seconds? In 240″? In 960″? In 10°? In 450°? In 78°?

5. How many nails in 3 quarters? In 7qr.? In 15qr.? In 2yd.? In 5yd.? In 13yd.?

6. How many pints in 8 gills? In 16gi.? In 32gi.? In 96gi.? In 8qt.? In 12qt.? In 2gal.? In 5gal.?

II. WEIGHTS.

1. TROY WEIGHT.

24 grains (gr.)	make 1 pennyweight.	dwt.
20 pennyweights	make 1 ounce.	oz.
12 ounces	make 1 pound.	lb.

The standard Troy pound[*] is equivalent to the weight of 22.79442 cubic inches of distilled water at its maximum density, the barometer standing at 30 inches; or to 22.8157 c. in. of water at 62° Fahrenheit, barometer at 30 inches.[†] It was copied by Captain Kater, in 1827, from the English Imperial Troy pound.

At the UNITED STATES MINT, the Troy ounce is adopted as the standard, and all weights are expressed in decimal multiples and submultiples of the ounce. Thus 95lb. 8oz. 15dwt. 15gr. of bullion, would be credited on the Mint books as 1148.65625oz.[‡]

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[*] The Troy pound was declared to be "the standard and Troy pound of the Mint of the United States, conformably to which the coinage thereof shall be regulated," by an act of Congress of 19th May, 1828. This is the only *direct* legislation in regard to the adoption of standards, but the joint resolution of June 14th, 1836, indirectly recognises the weights and measures used at the Custom Houses, as having been "adopted as standards."

[†] McCulloch.

[‡] Professor McCulloh, of the United States Mint.

2. APOTHECARIES' WEIGHT.

20 grains (gr.)	make 1 scruple.	℈.
3 scruples	make 1 dram.	ʒ.
8 drams	make 1 ounce.	℥.
12 ounces	make 1 pound.	℔.

The pound is the same as the pound Troy. This weight is used only in compounding medicines;—drugs are bought and sold by Avoirdupois Weight.

3. DIAMOND WEIGHT.

4 quarters	make 1 grain.
4 grains	make 1 carat.

The Diamond grain is $\frac{4}{5}$ of a Troy grain. The gold carat grain $= 2\frac{1}{5}$dwt.

4. AVOIRDUPOIS WEIGHT

10 grains	make 1 scruple.	sc.
3 scruples	make 1 dram.	dr.
16 drams	make 1 ounce.	oz.
16 ounces	make 1 pound.	lb.
28 pounds	make 1 quarter.	qr.
4 quarters	make 1 hundred-weight.	cwt.
20 hundred-weight	make 1 ton.	T.

The standard Avoirdupois pound is equivalent to 7000 Troy grains, or to the weight of 27.7274 c. in. of distilled water, at 62° Fahrenheit, barometer at 30 inches.*

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* The old English pound, which is said to have been the legal standard of weight, from the time of William the Conqueror to that of Henry VII., was derived from the weight of grains of wheat; 32 grains gathered from the middle of the ear, and well dried, made a pennyweight, 20 pennyweights an ounce, and 12 ounces a pound. Henry VII. altered this weight and introduced the present Troy pound, which is $\frac{3}{4}$ of an ounce heavier than the Saxon pound. The Avoirdupois pound was introduced by a statute of 24 Henry VIII.—*Brande.*

In many of the states, statutes have been enacted, fixing the ton at 2000lb., the hundred at 100lb., and the quarter at 25lb. But in the standard of the general government, the ton of 2240lb. with its subdivisions, is still retained.* Even where the legal ton is 2000lb., 2240lb. are often allowed in weighing bulky or cheap materials, such as iron, coal, plaster, &c.

5. FRENCH DECIMAL WEIGHT.

10 milligrammes	make 1 centigramme.
10 centigrammes	make 1 decigramme.
10 decigrammes	make 1 GRAMME.
10 grammes	make 1 decagramme.
10 decagrammes	make 1 hectogramme.
10 hectogrammes	make 1 kilogramme.
10 kilogrammes	make 1 myriagramme.
10 myriagrammes	make 1 quintal.
10 quintals	make 1 million, or bar.

1 gramme = 1.543402 grains, Troy.

The Gramme is a cubical centimetre.

The half kilogramme is called a *livre usuelle*, and the livre is subdivided as follows:

4 gros	make 1 once.
16 onces	make 1 livre usuelle.

EXAMPLES FOR THE PUPIL.

1. How many grains in 2 pennyweights? In 5dwt.? In 7dwt.? In 1oz.? In 5oz.? In 1lb.? In 3lb.?

2. How many dwt. in 48gr.? In 96gr.? In 288gr.? In 4oz.? In 10oz.? In 5lb.? In 7lb.?

3. How many scruples in 100 grains? In 240gr.? In 720gr.? In 8ʒ? In 17ʒ? In 1℥? In 5℥?

4. How many drams in 4oz. Avoirdupois? In 9oz.? In 13oz.? In 2lb.? In 5lb.?

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* See the different Tariff Acts.

5. How many ounces in 128dr. Avoirdupois? In 112dr.? In 192dr.? In 256dr.? In 4lb.? In 11lb.? In 3qr.?

III. CURRENCIES.

1. United States.

10 mills (m.)	make 1 cent.	ct.
10 cents	make 1 dime.	d.
0 dimes	make 1 dollar.	$.
10 dollars	make 1 eagle.	ea.

The gold and silver coins contain $\frac{9}{10}$ pure metal, and $\frac{1}{10}$ alloy. The alloy of gold is composed of about $\frac{1}{4}$ silver and $\frac{3}{4}$ copper, (not to exceed $\frac{1}{2}$ of silver); the alloy of silver is pure copper. The metal thus alloyed is called *standard*. The eagle contains 258 grains of standard gold, the dollar 412½ grains of standard silver, and the cent 168 grains of copper, and the multiples and subdivisions of all the coins, the same proportion. Previous to 1834, the eagle contained 270 grains, of which 22½ grains were alloy. By the act of Congress, June 28th, 1834, it was provided that all gold coins minted anterior to the 31st of July, of that year, should be receivable in all payments at the rate of 94.8 cents per pennyweight. The old eagle is therefore worth $10.665.

2. Great Britain.

4 farthings (qr.)	make 1 penny.	d.
12 pence	make 1 shilling.	s.
20 shillings	make 1 pound.	£.
21 shillings	make 1 guinea.	

A pound of silver of the English mint standard, contains 11oz. 2dwt. of pure silver, and 18dwt. of alloy. This pound is coined into 66 shillings. A shilling therefore weighs 87.27 grains, and contains 80.727 grains pure silver. The standard for gold is 11 parts fine gold and 1 part alloy. The sovereign weighs 123.274 grains, and contains 113.001 grains of pure gold. A sovereign of full weight is therefore worth $4.866.* The guinea and its subdivisions have not been coined since 1816.

3. FRANCE.

10 centimes (c.)	make 1 decime.	d.
10 decimes	make 1 franc.	f.

A franc is equal to 18$\frac{3}{5}$ cents in silver. Before the Revolution, the money was divided as follows:

12 deniers = 1 sol or sou.
20 sous = 1 livre tournois.
6 livres tournois = 1 ecu or crown.
4 crowns = 1 louis d'or.

The livre tournois was equivalent to 18$\frac{1}{5}$ cents.

4. SPAIN.

34 maravedis	make 1 rial.
10 rials of plate	make 1 dollar.

1 rial of plate = 2 rials vellon.

The Spanish dollar of silver, is of the same value as the American dollar; the dollar of gold is worth 98 cents. The coin of plate is silver; vellon is the old copper coin of Castile, and is of but half the value.

5. PORTUGAL.

1000 rees	make 1 milree.
400 rees	make 1 crusado of exchange.

The milree is equal to $1.31 in gold, or $1.12 in silver.

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* In 1838 a dispute arose in the settlement of an account, which was submitted to arbitration. The charges were all made in English money, and the point in dispute was the value of the pound sterling, in our own currency. It was decided by the referees, to be $4.858.—*Hunt's Merchants' Magazine.*

At the Custom House, the sovereign is estimated at $4.84; the pound of the British Provinces, Nova Scotia, New Brunswick, Newfoundland, and Canada, at $4.00; the pound of Jamaica, Honduras, and Turk's Island, at $3.00; the pound of Nassau, at $2.50.

6. Holland.

100 cents	make 1 florin.

The florin is equal to $0.40 in silver. Accounts are still sometimes kept in Flemish money, the denominations of which are,

6 pennings	make 1 groat.
2 groats	make 1 stiver.
6 stivers	make 1 schilling.
8⅓ schillings	make 1 florin.
6 florins	make 1 pound.

7. Denmark.

16 skillings	make 1 mark.
6 marks	make 1 dollar.

The silver dollar, coined before 1813, was equivalent to $1.04. The new, or Rigsbank dollar, is worth but 52 cents.

8. Hamburgh.

12 pfenings	make 1 sol, or schilling, lubs.
16 schillings lubs	make 1 marc.
3 marcs	make 1 rix dollar.

The marc current is about 29 cents; the marc banco, about 35 cents, but the currency is exceedingly fluctuating. Lubs denotes the money of Lubec. In exchanges, accounts are frequently kept in Flemish money.

9. Norway.

120 skillings make 1 species dollar.

The species dollar of silver is equivalent to $1.05.

10. Sweden.

12 rundstycks	make 1 skilling.
48 skillings	make 1 rix dollar.

The rix dollar banco = 40 cents.

11. Russia.

100 copecks	make 1 rouble.

The bank rouble = about 22 cents. The silver rouble = 75 cents. The gold half Imperial, or 5 rouble piece = $3.94.

12. PRUSSIA.

12 pfenings	make 1 silver groschen.
30 silver groschen	make 1 thaler.

The thaler of silver is equivalent to $69\frac{2}{5}$ cents. Gold coins are bought and sold as merchandise.

13. AUSTRIA.

4 pfenings	make 1 creutzer.
60 creutzers	make 1 florin.
$1\frac{1}{2}$ florins	make 1 rix dollar of account.
2 florins	make 1 rix dollar specie.

The silver rix dollar is equal to $96\frac{2}{5}$ cents. Gold is not legal tender, but circulates only as merchandise.

14. NAPLES.

10 grani	make 1 carlino.
10 carlini	make 1 ducato.

The ducat = 80 cents in silver.

15. SICILY.

20 grani	make 1 taro.
10 tari	make 1 ducato.
8 ducati	make 1 oncia.

The ducat is of the same value as that of Naples.

16. VENICE AND GENOA.

100 centesimi make 1 lira Italiana.

The lira Italiana is intended to be equivalent in value to the franc.

17. LEGHORN.

12 denari	make 1 soldo.
20 soldi	make 1 pezzo.

1 pezzo = $5\frac{3}{4}$ lire.

The pezzo is equivalent to 90 cents in silver.

18. TURKEY.

3 aspers make 1 para.
40 paras make 1 piastre.

The piastre is worth about 4 cents, but its value is exceedingly variable.

19. EAST INDIES.

4 cowries make 1 gunda.
20 gundas make 1 punn.
3 pice make 1 punn.
4 punns make 1 anna.
16 annas make 1 rupee.
16 rupees make 1 gold mohur.

The gold mohur is equivalent to $8.08; the Sicca rupee of silver, to 47½ cents; the current rupee, to about 44 cents. The cowrie is a species of shell. A lac of rupees is 100000 rupees. A crore is 100 lacs.

20. CANTON.

10 cash make 1 candarine.
10 candarines make 1 mace.
10 mace make 1 tael.

The tael is a weight, about equivalent to 1oz. 4dwt. 4gr. Troy. The tael of silver is usually estimated at $1.48.

EXAMPLES FOR THE PUPIL.

1. How many cents in 4 dimes? In 6 dimes and 3 cents? In $2? In $5 8d. 6ct.? In $3 2d. 5ct.?

2. How many farthings in 5d.? In 7d.? In 1s.? In 5s. 2d.? In 2s. 3d. 3qr.? In 9d. 2qr.?

3. How many shillings in 36d.? In 84d.? In 228d.? In £4? In £7 6s.? In £18 18s.?

4. How many centimes in 1 franc? In 2f. 7d.? In 9f. 4d.? In 3d. 8c.? In 7f. 7d. 7c.

5. How many rees in 5 milrees? In 7 milrees 850 rees? In 4 milrees 2 crusados?

60. REDUCTION OF COMPOUND NUMBERS.

EXAMPLES FOR THE BOARD.

r. *yd.* *ft.* *in.*
27 4 2 11
5.5
135
135
148.5 yd.
4 yd.
152.5 yd.
3
457.5 ft.
2 ft.
459.5 ft.
12
5514.0 ft.
11 in.
5525 in.

1. How many inches in 27r. 4yd. 2ft. 11in.? As there are 5.5 yards in 1 rod, in 27 rods there will be 148.5 yards. In 27r. 4yd. there will then be 152.5yd. As 3ft. make 1yd., in 152.5yd. there will be 457.5ft. Adding the 2ft., we obtain 459.5ft. As 12in. make 1ft., in 459.5ft. there are 5514in. Adding the 11in., we have 5525in. for the answer. Hence, to reduce the higher denominations of a compound number to their value in a lower denomination, we have the following

RULE:

Commence with the highest denomination, and multiply each denomination by the number required of the next lower to compose it, adding to the product the number, (if any,) already in that denomination. Thus proceed until you have reached the lowest denomination sought.

12)5723
3)476 ft. 11 in.
5.5)158 yd. 2 ft.
28 r. 4 yd.

2. How many r., yd., ft., and in. in 5723 inches? 5723in. $=\frac{5723}{12}$ft. $=$476ft. 11in. 476ft. $=\frac{476}{3}$yd. $=$ 158yd. 2ft. 158yd. $=\frac{158.0}{5.5}$r. $=$ 28r. 4yd. The answer is, therefore, 28r. 4yd. 2ft. 11in. Hence, to reduce the lower denominations of a compound number to their value in higher denominations, we have the following

RULE:

Divide each denomination by the number required of it to make 1 *of the next higher, and give to each remainder the name of the dividend from which it is derived.*

61. EXAMPLES FOR THE PUPIL.

1. In 41chal. 19bu. of coal, how many pecks?
Ans. 5980*pk.*

2. How many cwt., &c., in 1870953gr.?
Ans. 2*cwt.* 19*lb.* 9*oz.* 13*dr.* 3*gr.*

3. How many ℔, &c., in 24753gr. Troy?

4. Reduce 195℔ 11℥ 3ʒ to grains.
Ans. 1128660*gr.*

5. Reduce 69741895 inches to miles.
Ans. 1100*m.* 5*fur.* 31*r.* 4*yd.* 1*ft.* 1*in.*

6. Reduce $\frac{7}{16}$cwt. to qr., lb., &c.*
Ans. 1*qr.* 21*lb.*

7. Reduce £9.481 to £, s., &c.
Ans. £9 9*s.* 7.44*d.*

8. Reduce 7964 gills to gallons, &c.
Ans. 248*gal.* 3*qt.* 1*pt.*

9. Reduce $\frac{49}{53}$yr. to days, &c.
Ans. 337*dy.* 10*h.* 52*min.* 4$\frac{28}{53}$*s.*

10. Reduce 51$\frac{3}{11}$Cong. to Cong., O., &c.
Ans. 51*Cong.* 2*O.* 2*f*℥ 7*f*ʒ 16$\frac{4}{11}$♏.

11. Reduce 8753in. to yd., qr., &c.

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* The reduction of fractions and decimals is performed in the same way as reduction of whole numbers, by multiplying or dividing, as the question may require.

12. How many acres in a piece of land that is 15r. 4yd. long, and 14r. 2yd. 2ft. wide?

Ans. 62020.5*sq. ft.* = 1*A.* 1*R.* 27*r.* 24*yd.* 3*ft.* 108*in.*

13. How many cords in a pile of wood 21.5ft. long, 8.3ft. wide, and 9.7ft. high?

Ans. 1730.965*c. ft.* = 13*c.* 4*c. ft.* 2.965*c. ft.*

14. Reduce 4$\frac{2}{9}$yr. to seconds. *Ans.* 133152000*sec.*

15. What is the value of £197 7s. 6d. in Federal Money, estimating the penny at 2 cents?

Ans. $947.40.

16. What is the value of 6971 roubles 25 copecks, at $\frac{3}{4}$ of a cent per copeck? *Ans.* 5228.43\frac{3}{4}$.

17. Reduce $\frac{7}{11}$ of a mile to fur., r., &c.

18. Reduce 10 pounds 17 shillings 10 groats Flemish, to dollars and cents, at a cent a groat. *Ans.* $26.14.

19. Reduce 172843'''' to feet, primes, &c.

62. TO REDUCE LOWER DENOMINATIONS TO THE FRACTION OF A HIGHER.

EXAMPLE FOR THE BOARD.

What part of a £ is 2s. 7d. 3qr.?

2s. 7d. 3qr. = 127qr. = $\frac{127}{4}$d. = $\frac{127}{4}$ of $\frac{1}{12}$s. = $\frac{127}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$£ = $\frac{127}{960}$£.

RULE.

First reduce the given sum to the lowest denomination mentioned. With this result, make a fraction of the next higher denomination, then a compound fraction of the next higher, and so proceed until you reach the denomination required.

63. EXAMPLES FOR THE PUPIL.

1. Reduce 9s. 3d. 2qr. to the fraction of a £.

Ans. £$\frac{223}{480}$

2. Reduce 3qt. 1pt. 2gi. to the fraction of a gallon.

3. Reduce 13h. 11m. 11sec. to the fraction of a day. *Ans.* $\frac{47471}{86400}$*dy.*

4. Reduce 4yd. 10in. to the fraction of a rod.

5. Reduce 7O. 6f℥ to the fraction of a gallon. *Ans.* $\frac{902}{1024}$ *Cong.*

6. Reduce 7s. 6d. to the fraction of a guinea. *Ans.* $\frac{90}{252}$*guin.*

7. Reduce 3qr. 3na. to the fraction of a yard.

8. What part of 7lb. is 13oz. 14dr.? *Ans.* $\frac{111}{896}$.

64. To Reduce Lower Denominations to the Decimal of a Higher.

EXAMPLE FOR THE BOARD.

Reduce 10oz. 15dwt. 22gr. to the decimal of a ℔. 22gr. $=\frac{22}{24}$ dwt. $=.91\dot{6}$dwt. Therefore, 15dwt. 22gr. $=15.91\dot{6}$dwt. $=\frac{15.91\dot{6}}{20}$oz. $=.7958\dot{3}$oz. Then, 10oz. 15dwt. 22gr. $=10.7958\dot{3}$oz. $=\frac{10.7958\dot{3}}{12}$℔ $=.899652\dot{7}$lb.

RULE.

Commence with the lowest denomination, annex decimal 0s, and reduce each denomination to the decimal of the next higher, by dividing as in ordinary reduction.

65. EXAMPLES FOR THE PUPIL.

In the following examples, the numbers are all to be reduced to the decimal of the highest denomination mentioned.

1. Add £11 11s. 11d.; £4 3s. 7d.; 2s. 6d. 3qr.; and 9s. 6d. *Ans.* £16.378125 = £16 7*s.* 6*d.* 3*qr.*

2. Add 6gal. 3qt. 1pt. 3gi.; 2qt. 1pt. 1gi.; 5gal. 3gi.; and 4gal. 1qt. *Ans.* 16*gal.* 3*qt.* 1*pt.* 3*gi.*

3. From 19T. 10cwt. subtract 15cwt. 3qr. 14lb. *Ans.* 18*T.* 14*cwt.* 14*lb.*

4. From 7Cong. 7O. subtract 1Cong. 7fʒ.
Ans. 6 *Cong.* 6 *O.* 15*f*℥ 1*f*ʒ.

5. Divide 17℔ 6℥ 4ʒ by 5.
Ans. $3.508\dot{3}$℔ = 3℔ 6℥ 2℈ 8*gr.*

6. Reduce £591 9s. 3d. to Federal Money, at $4.85 per £. *Ans.* $2868.59 +.

7. Reduce 28 ducats 9 carlins 7 grains of Naples to dollars and cents, at 83ct. per ducat. *Ans.* $24.04.

8. Reduce 28C. 7C. ft. 4c. ft. to cords and decimals of a cord. *Ans.* 28.90625 *C.*

9. What is the value of 94 francs 79 centimes, at 18¾ct. per franc? *Ans.* $\$17.77\frac{5}{16}$.

10. What is the value of 75 milrees 2 crusados, at $1.15 per milree? *Ans.* $87.17.

66. COMPOUND ADDITION.

EXAMPLE FOR THE BOARD.

£.	*s.*	*d.*	*qr.*
20	16	9	3
11	0	7	1
9	19	11	2
41	17	4	2

What is the sum of £20 16s. 9d. 3qr.; £11 7d. 1qr.; and £9 19s. 11d. 2qr.? Writing the numbers as in Simple Addition, placing those of the same denomination in the same perpendicular column, I commence with the column of farthings, and find its sum, 6qr. = 1d. 2qr. I write the 2qr. beneath the column of qr., and carry the 1d. to the column of d. The sum of the column of d. is 28d. = 2s. 4d. Write the 4d. beneath the column of d., and carry 2s. to the column of s. There are 37s. = £1 17s. Write 17 beneath the column of s., and carry £1 to the column of £. The sum of the £ is 41, which is written down, as there is no higher denomination.

RULE.

Write the numbers as in Simple Addition, placing those of the same denomination in the same vertical column.

Commence at the right hand to add, and reduce the sum of each column to the next higher denomination. Write the remainder underneath, and carry the quotient to the next column.

67. EXAMPLES FOR THE PUPIL.

1. Add 9qr. 1C. 2bu. 3pk.; 4qr. 1bu. 1pk. 1gal.; 3bu. 3pk. 1qt.; and 5qr. 2pk., and reduce the result to quarts.

2. Add 3Cong. 7O. 15f℥ 7fʒ 59♏; 7Cong. 3f℥; 4O. 9f℥ 4fʒ; and 2Cong. 4fʒ 1♏.
Ans. 13*Cong.* 4*O.* 13*f℥.*

3. Add 116gal. 3qt. 1pt.; 59gal. 1pt. 3gi.; 76gal. 2qt.; and 291gal. 1qt. 1pt. 1gi. *Ans.* 544*gal.*

4. Add 14T. 9cwt. 3qr. 17lb.; 9T. 1qr. 25lb. 5oz.; 16lb. 3oz. 14dr.; and 17T. 18cwt. 1qr. 27lb. 2dr.

5. Add £19 7s. 6d. 3qr.; £276 19s. 11d.; 14s. 3qr.; £278 13s.; and £190 5s. 5d. 2qr.

6. Add 5℔ 11℥ 7ʒ 2℈ 18gr.; 7℔ 14gr.; 3℥ 3ʒ; and 15℔ 4℥ 19gr., and reduce the ℥, &c., to the fraction of a ℔. *Ans.* $28\frac{3571}{5760}$℔.

7. Add 55yd. 3qr. 3na.; 7yd. 1na. 1in.; 4yd. 2qr. 1na.; 63yd. 2na.; and 175yd. 1qr. 2na. 2in., and reduce the result to decimal yards. *Ans.* 306.14583*yd.*

8. Add 3C. 7C. ft. 15c. ft. 1719c. in.; 14C. 3c. ft. 96c. in.; 2C. ft. 14c. ft.; 47C. 6C. ft.; and 19C.

9. Add 9sq. m. 627A. 3R. 30r.; 3sq. m. 248A. 39r.; 41sq. m. 27r.; and 163A. 2R. 12r., and reduce the result to rods. *Ans.* 5593588*r.*

10. Add 17m. 7fur. 39r. 4yd. 2ft. 11in.; 19m. 3fur. 3yd.; 6fur. 18r. 1ft. 9in.; and 41m. 24r. 2ft. 7in.

11. How many deniers in 4 Louis d'ors 1 ecu 5 livres +

13 Louis 2 livres 19 sous 10 deniers + 2 livres 7 sous + 4 livres 13 sous 9 deniers? *Ans.* 102967*den.*

12. Add 19 ducats 8 carlins 7 grains; 12d. 3c. 5gr.; 9c. 8gr.; and 29d. 2c., and find their value at 81cts. per ducat. *Ans.* $50.54.

13. Add 19ft. 11′ 8″ 5‴; 3ft. 4″ 9‴; 10′ 10″; 8ft. 6′; 15ft.; and 29ft. 2′ 7″ 11‴.

68. Compound Subtraction.

EXAMPLE FOR THE BOARD.

bu.	*pk.*	*qt.*	*pt.*
39	1	3	1
19	2	5	1
19	2	6	0

From 39bu. 1pk. 3qt. 1pt., take 19bu. 2pk. 5qt. 1pt. Commencing at the right hand, as in Simple Subtraction, I say, 1pt. from 1pt. leaves 0pt. 5qt. cannot be taken from 3qt. I therefore increase the upper number by 8qt., and the lower number by 1pk., and say, 5qt. from 11qt. leaves 6qt., 3pk. from 5pk. leaves 2pk., and 20bu. from 39bu. leaves 19bu.

RULE.

Write the numbers as in Compound Addition, and commence at the right hand to subtract. If the number representing any denomination in the minuend, is less than that in the subtrahend, increase both the minuend and the subtrahend by one of the next higher denomination, and proceed as in ordinary subtraction.

69. EXAMPLES FOR THE PUPIL.

1. From 28chal. 8bu. 1pk., take 18chal. 31bu. 2pk. 7qt

2. What is the difference between 187gal. 3qt. 1pt. 3gi. and 216gal. 2gi.? *Ans.* 28*gal.* 3*gi.*

3. Subtract 7yr. 191dy. 18h. 59m. 2sec. from 8yr. 32dy 17h. *Ans.* 205*dy.* 22*h.* 58*sec.*

4. From 19Cong. subtract 7O. 11f℥ 3fʒ 41♏, and reduce the remainder to ♏. *Ans.* 1108099♏.

5. A grocer purchased 19cwt. 1qr. of sugar. How much had he left after selling 11cwt. 2qr. 17lb. 12oz.?

6. A man sold his horse for £40, which was £9 8s. 6d. more than it cost him. What did he give for the horse?

7. Subtract 9℔ 7℥ 7ʒ 1℈ 11gr. from 10℔ 3ʒ 9gr.

8. Subtract 19yd. 3qr. 3na. from 117yd. 1qr. 2na.

9. A ship left port November 29th, 1841, and returned August 18th, 1844. How long was her voyage?
Ans. 2*yr.* 8*mo.* 19*dy.*

10. From a lot of land containing 490 acres, 163A. 3R. 17r. were sold. How much was left?

11. From a board measuring 13ft. 9′ 11″, a piece was sawed containing 4ft. 0′ 3″ 8‴. How much was left?

12. How many seconds elapsed from 20min. past 5 A. M. 8th mo. 18, 1829, to 17min. past 3 P. M. 5th mo. 3, 1831? *Ans.* 623*dy.* 9*h.* 57*min.* = 53863020*sec.*

13. Subtract 9 marcs 13 schillings 11 pfennings Hamburg, from 10m. 3sch., and reduce the remainder to the fraction of a marc. *Ans.* $\frac{61}{192}$*marc.*

14. From 7m. 3fur. 9ch., subtract 7fur. 3p. 18 l., and reduce the remainder to miles and decimals of a mile.
Ans. 6.600875*m.*

70. Compound Multiplication.

EXAMPLE FOR THE BOARD.

yd.	*ft.*	*in.*
9	2	11
		15
149	1	9

Multiply 9d. 2ft. 11in. by 15. Multiplying the inches by 15, we find 165in. = 13ft. 9in. Write 9 in the place of inches, below the line, and carry 13ft. 15 times 2ft. = 30ft., and 13ft. to carry, make 43ft. = 14yd. 1ft. Write 1ft. in the column of feet, and carry 14yd. 15 times 9yd. = 135yd., and 14yd. to be carried, make 149yd.

RULE.

Commence with the smallest denomination, and reduce each PRODUCT *to the next higher denomination, writing the remainder underneath, and carrying the quotient as in Compound Addition.*

71. EXAMPLES FOR THE PUPIL.

1. Multiply 13yr. 79dy. 18h. 43min. 12sec. by 43.
Ans. 568*yr.* 145*dy.* 12*h.* 57*m.* 36*s.*

2. Multiply 19m. 7fur. 4r. 4yd. 11in. by 81.
Ans. 1611*m.* 27*r.* 2*yd.* 9*in.*

3. Multiply £96 11s. 11d. 1qr. by 57.
Ans. £5506 5*d.* 1*qr.*

4. Multiply 41yd. 2qr. 3na. 1in. by 79.
Ans. 3295*yd.* 2*qr.* $\frac{1}{4}$*in.*

5. Multiply 28T. 18cwt. 2qr. 13lb. by 47.

6. Multiply 781gal. 1gi. by 46.

7. Multiply 83Cong. 7f℥ 41♏ by 23.

8. How many seconds in 13 times 4yr. 18d. 13h.?
Ans. 1660698000*sec.*

9. How many feet, primes, &c., in 86 times 3 9′ 0″ 4‴?

10. Multiply 7m. 3fur. 5ch. by 17, and reduce the fur. and ch. to the fraction of a mile. *Ans.* 126$\frac{7}{16}$*m.*

11. Multiply 19 4′ 2″ 11‴ by 29, and reduce the result to ‴. *Ans.* 969847‴.

72. COMPOUND DIVISION.

EXAMPLE FOR THE BOARD.

yd.	*ft.*	*in.*
5)19	2	9
3	2	11$\frac{2}{5}$

Divide 19yd. 2ft. 9in. by 5. Dividing the 19yd., we have a quotient of 3yd., with a remainder of 4yd. The undivided 4yd. are equivalent to 12ft., which, added to the 2ft., make 14ft. 14ft. ÷ 5 = 2ft., with a remainder of 4ft. The undivided 4ft.

are equal to 48 inches, which, added to the 9 inches, make 57 inches. 57in. ÷ 5 = $11\frac{2}{5}$in.

RULE.

First divide the highest denomination. Whenever there is any remainder, reduce it to the next lower denomination, and add it to the number, (if any,) already in that denomination. Then divide as before, and so proceed until the division is completed. If both the divisor and the dividend are compound numbers, reduce them to the same denomination, and divide as in simple division.

73. EXAMPLES FOR THE PUPIL.

1. What is the price of a qr. of sugar, at £2 8s. per cwt.? *Ans.* 10*s.* 9*d.*

2. Divide 93gal. 1qt. by 11; by 19. *Ans.* 8*gal.* $1\frac{10}{11}$*qt.*; 4*gal.* $3\frac{12}{19}$*qt.*

3. How many barrels, each holding 2bu. 3qt., can be filled with 41bu. 2pk. of apples? *Ans.* $19\frac{55}{67}$*bbl.*

4. Divide 119Cong. 15f℥ 3fʒ by 28.

5. Divide 79yd. 1qr. 1na. by 4yd. 3qr. *Ans.* $16\frac{53}{76}$.

6. Divide $\frac{7}{8}$ of a mile by 3ft. 9in. *Ans.* 1232.

7. Divide 4.76yr. by 3.9dy. *Ans.* $445\frac{19}{39}$.

8. Divide 196℔ 11℥ 2℈ by 24. *Ans.* 8℔ 2℥ 3ʒ $2\frac{1}{12}$℈.

9. Divide $27\frac{5}{7}$T. by $\frac{3}{7}$cwt. *Ans.* $1293\frac{1}{3}$.

10. Divide 37 6′ 6″ by 9; by 13.

11. Divide 49 × £6.725 by 31; by 53.

74. DUODECIMALS.

DUODECIMALS are compound numbers, in which each of the lower denominations is $\frac{1}{12}$ of the one above it. They are used in square, linear, and cubic measure.

The denominations of duodecimals are, the foot, or unit; the inch, or prime, marked ($'$); the second, or twelfth of a prime ($''$); the third, or twelfth of a second ($'''$); the fourth ($''''$), and so on. These marks are called *indices.*

Duodecimals are added and subtracted in the same manner as other compound numbers; but multiplication and division, when both numbers contain duodecimals, present some difficulties.

The product of units by units is units; units by primes, give primes; units by seconds, give seconds, &c. Primes by primes, give seconds; primes by seconds, give thirds, &c. Generally, to the product of any two denominations, we must annex as many indices as there are in both factors. For example, $5'' \times 5''''$, is the same as $\frac{5}{12}$ of $\frac{1}{12} \times \frac{5}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$. In one factor, the denominator 12 is contained twice; in the other, four times. Then, in their product, it will be contained six times, which will be represented by $25''''''$. For a similar reason, in division, we subtract the indices of the divisor from those of the dividend. Then, $32'''' \div 8'' = 4''$.

75. EXAMPLES FOR THE BOARD.

$$\begin{array}{rrrrr}
2 & 3' & 7'' & & \\
 & & 9 & 5' & 11'' \\
\hline
20 & 8' & 3'' & & \\
 & 11' & 5'' & 11''' & \\
 & 2' & 1'' & 3''' & 5'''' \\
\hline
21 & 9' & 10'' & 2''' & 5''''
\end{array}$$

What is the product of 2 3' 7'' by 9 5' 11''? We write the units' figure of the multiplier under the right hand figure of the multiplicand, because the product of any denomination by units is of the same value. The product by 12ths will be 12 times smaller than that by units, the product by seconds 12 times smaller than that by primes, and so on. We therefore write the first figure of each product under the figure by which we multiply, as in simple multiplication. When any product, or the sum of any column below units, is greater than 12, we reduce it to its value in higher denominations, and carry the quotient to the next column at the left.

Divide 21 9′ 10″ 2‴ 5⁗ by 9 5′ 11″.

```
9  5′  11″)21   9′  10″  2‴  5⁗(2 3′ 7″
           18  11′  10″
           ───────────
            2  10′   0″  2‴
            2   4′   5″  9‴
            ──────────────
                5′   6″  5‴  5⁗
                5′   6″  5‴  5⁗
                ─────────────────
```

9 (units) is contained in 21 (units) 2 (units) times. Multiplying and subtracting, we have a remainder, 2 10′ 0″.—Bringing down the 2‴, we reduce 2 10′ to primes, and find that 34′ contains 9, 3′ times. Multiplying and subtracting as before, we have a remainder 5′ 6″ 5‴, to which we annex the 5⁗ of the dividend. 5′ 6″ = 66″, which contains 9, 7″ times. Multiplying by 7″, there is no remainder; the quotient is therefore 2 3′ 7″. These operations are so analogous to those of Simple Multiplication and Division, that they require no separate rules.

76. EXAMPLES FOR THE PUPIL.

1. How many feet, &c., are there in 6 boards, each of which is 13 11′ long, and 1 9′ wide? *Ans.* 146 1′ 6″.

2. How many yards of carpeting that is 2 10′ wide, will cover a floor 16 feet long and 13 6′ wide? *Ans.* $25\frac{7}{17}$*yd.*

3. A carpenter has 4 boards, the first of which is 12ft. long and 2ft. wide; the second, 11 9′ long and 1 9′ wide; the third, 12 7′ long, and 1 11′ wide, and the fourth 13 2′ long and 1 8′ wide. How many square feet are there in the whole? *Ans.* $90\frac{5}{8}$*sq. ft.*

4. How many cubic feet in a wall 19ft. 6in. long, 2ft. 3in. thick, and 4ft. 6in. high? *Ans.* $197\frac{7}{16}$*c. ft.*

5. How many cubic feet of wood in a pile 16 3′ long, 4 6′ wide, and 8 feet high?

6. The number of square feet, &c., in a floor, is 166 9′ 6″ 3‴ 9⁗, and the length of the room is 17 11′ 3″. What is the breadth? *Ans.* 9 3′ 7″.

7. How much plastering is there in the ceiling of a room 21 4′ long and 18 7′ 6″ wide? *Ans.* 397 4′.

8. How many square feet of glass in 8 windows, of 12 panes each, each pane being 1 1′ long and 9′ wide? *Ans.* 78*sq. ft.*

9. Add 4 0′ 3″, 18 5′, and 3 8′ 2″, subtract from the sum 13 5′ 11″, multiply the remainder by 5 times 4 2′ 4″, and divide the product by 4 2′ 6″. *Ans.* 62 11′.

XI. REVIEW.

77. MISCELLANEOUS EXAMPLES.

1. John Adams was born October 30, 1735, and died July 4, 1826. What was his age at the time of his death?

2. Roger Sherman was born April 19, 1721, and died when he was 72 years 3 months and 4 days old. On what day did he die?

3. Benjamin Franklin died April 17, 1790, aged 84 years and 3 months. When was he born?

4. In a certain farm that is 142.5 rods long, and $119\frac{3}{4}$ rods wide, there is a pond containing $13\frac{2}{7}$ acres. How many acres are not covered by water? *Ans.* $93\frac{657}{1792}$.

5. The pyramid of Cheops measures 763.4 feet on each side of its base. How many acres does it cover? *Ans.* 13*A.* 1*R.* 20*r.* 18*yd.* 2.56*ft.*

6. A mason builds a wall 14 11′ 3″ long, 2ft. thick, and 4 6′ high. What will he receive for the whole, at $0.75 per foot? *Ans.* $100.83.

7. What would be the weight of a gold wire that would reach from Boston to Liverpool, estimating the distance at

2997m. 3fur. 28r., and the weight of the wire at $\frac{1}{1800}$gr. avoirdupois, per foot? *Ans.* 1*lb.* 2*oz.* 5*dr.* 3*gr.*

8. The wheel of a carriage measures 13.625 feet in circumference. How many times will it revolve in going from Philadelphia to New York, the distance being 89 miles? *Ans.* 34443.6 + *times.*

9. What is the cost of 9T. 11cwt. 3qr. of sugar, at 5d. 3qr. per pound?

10. What part of £279 11s. is £31 9s. $6\frac{3}{4}$d.? *Ans.* $\frac{10073}{89456}$.

11. Out of 120 million passengers carried on the English railroads, from 1840 to 1845, both inclusive, only 66 were killed, and 324 were wounded.[*] Reduce to the lowest terms, the fractions which represent the part of the entire number carried, which were killed, and which were wounded. *Ans.* $\frac{11}{20000000}$ *killed;* $\frac{27}{10000000}$ *wounded.*

12. A grocer purchased $19\frac{4}{7}$cwt. of sugar, at $6\frac{1}{2}$ cents a pound, and sold the whole for $156.75. How much did he gain by the sale?

13. What is the least common multiple of 130, 546, 156, and 182?

14. What is the greatest common divisor of 111, 222, 333, 444, 555, 666, 777, 888, 999999, and 74?

15. The length of wave corresponding to extreme red light, is .0000266 of an inch.[†] How many such waves would there be in a foot? *Ans.* 451128 *nearly.*

16. If $4\frac{7}{9}$ yards of calico cost $5\frac{1}{3}$ shillings, what is the price of 1 yard? *Ans.* $1\frac{5}{43}$*s.*

17. How many acres of land in a field that is $73\frac{1}{4}$ rods long, and $49\frac{1}{3}$ rods wide? *Ans.* 22*A.* 2*R.* $13\frac{2}{3}$*r.*

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[*] Gillespie.

[†] Lloyd.

18. How many rays of extreme violet light would there be in 6 inches, the length of each wave being .0000167 of an inch? *Ans.* 359281.4 +.

19. If $13\frac{7}{8}$ yards of broadcloth cost \$$62\frac{8}{9}$, what will be the cost of 1 yard? Of $7\frac{9}{10}$ yards? *Ans.* \$4.53; \$35.81.

20. How many arithmetics that are worth \$7.50 per dozen, can I buy for \$$10\frac{5}{8}$?

21. Reduce to a common denominator $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{1}{5}$ of $\frac{4}{7}$, $\frac{2}{5}$ of $\frac{11}{21}$, $\frac{97}{315}$, and $\frac{76}{231}$.

22. The diameter of the sun is 882000 miles.* Reduce this quantity to feet. *Ans.* 4656960000*ft.*

23. How much must I give for 49T. 5cwt. 1qr. of flour, at \$4.875 per barrel of 196 pounds? *Ans.* \$2744.625.

24. A farmer sold $5\frac{3}{4}$ tons of hay, at \$4.50 per bundle, each bundle containing 5cwt. How many bundles were there, and what did he receive for the whole?

25. A., B., and C. traded in company. A. contributed \$19000, B. \$18500, and C. \$25750. What was the whole stock in trade, and what part of the gain should each man receive? *Ans.* *A.* $\frac{76}{253}$; *B.* $\frac{74}{253}$; *C.* $\frac{103}{253}$.

26. The entire annual consumption of gas in London and its environs, is estimated at 3000000000c. ft., and the light produced by the combustion may be considered as equal to what would be obtained from 160000000lb. of tallow candles.† According to this estimate, how many feet of the gas would be equivalent in illuminating power to a pound of the candles?

27. How many potatoes, at 25 cents a bushel, must be given in exchange for $163\frac{3}{4}$ pounds of sugar, at $9\frac{1}{2}$ cents a pound? *Ans.* 62.225*bu.*

28. A. can do a piece of work in 8 days, and B. can do

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* Carpenter.

† Parnell.

it in 12 days. What part can they both do in 1 day, and how long will it take them both to do the whole?

29. It is said that the Emperor Nero paid 6000 sestertia for two glass cups with handles, on account of their perfect transparence.* What would be their value in Federal Money, estimating the sestertium at $38.75?

30. A merchant bought 17 bales of sheeting, each bale containing 18 pieces, and each piece $32\frac{1}{4}$ yards. How much did he give for the whole, at $11\frac{1}{2}$ cents a yard?
Ans. $1134.87\frac{3}{4}$.

31. How many men can build a house in 1 day, that 25 men can build in 73 days?

32. A labourer's wages are $1.75 per day. How many days must he work to pay for pasturing his cow $39\frac{1}{2}$ weeks, at $37\frac{1}{2}$ cents per week? *Ans.* $8\frac{13}{28}$*dy.*

33. How much sugar would be yielded in a single season, by a forest of 1000 maples, the average quantity of sap obtained from each being $17\frac{1}{7}$ gallons, 30 gallons of sap making 7lb. of sugar? *Ans.* 4000*lb.*

34. How many bottles, each holding $1\frac{1}{2}$ pints, may be filled from a cask containing $127\frac{1}{8}$ gallons of wine?

35. How many times will a wheel that is $12\frac{1}{2}$ feet in circumference, revolve in going from Boston to Providence, the distance being $39\frac{1}{3}$ miles? *Ans.* 16614.4 *times.*

36. A confectioner in London returned £30000 as his annual income.† Reduce the amount to Federal Money, estimating a penny as equivalent to 2 cents.

37. How much must a man spend a day, in order to use all his income, which is $1000 per annum?
Ans. $2.73\frac{71}{73}$.

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* Parnell.

† Colman.

38. Bought a house lot for $950.50. What is the value of $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{5}{9}$ of the lot? *Ans.* $198.02.

39. How much cloth is there in a piece worth \$127$\frac{3}{8}$, at \$4$\frac{1}{8}$ per yard?

40. A farmer in Lincolnshire raised 18000bu. of wheat in one year.* What was its value, at 4s. 6d. per bushel? *Ans.* £4050.

41. A merchant bought 56 barrels of flour for $238, and paid for transportation $14.00; storage, $4.75; drayage and other expenses, $5.00. At what price must he sell it per barrel, to gain $18.25 on the whole?

42. If $\frac{4}{5}$ of a ton of hay cost \$11$\frac{1}{5}$, what will 4$\frac{5}{9}$ tons cost? *Ans.* \$63$\frac{7}{9}$.

43. At what rate per hour is a person carried at the equator, by the diurnal revolution of the earth, the equatorial circumference being 24895.512 miles?† At what rate per minute?

44. How much must I give for 84$\frac{1}{3}$ gallons of vinegar, at 6$\frac{1}{4}$ cents a quart? *Ans.* \$21.08$\frac{1}{3}$.

45. At the battle of the Granicus, the Persian army amounted to 610000 foot, and 20000 horse; the Macedonian army, to 30000 foot and 5000 horse.‡ How many Persians were opposed to each Macedonian?

46. What would 96$\frac{7}{8}$lb. of pork cost, at 7$\frac{3}{4}$ct. per lb.? *Ans.* $7.51.

47. How long will it take 1 man to build a wall that 15 men can build in 4$\frac{11}{18}$ days?

48. How many square feet in a board 2ft. 3in. wide, and 11ft. 7in. long? *Ans.* 26$\frac{1}{16}$*sq. ft.*

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* Colman. † Downes. ‡ Nesbit.

49. Estimating the mean distance of the sun at 95000000 miles, and the diameter of the earth at 7924.5 miles, how many of the earth's diameters would reach to the sun?

50. The greater of two numbers is $87\frac{1}{9}$, and their difference is 7.625. What is the less?

51. The sum of two numbers is $96\frac{3}{17}$, and one of the numbers is $18\frac{5}{11}$; what is the other?

52. If 5.29 francs are worth $1.00, what is the value of 688.31 francs? *Ans.* $130.11.

53. In England, in political and ecclesiastical affairs, the year is considered as commencing on Lady-Day, which is the 25th of March. From Lady-Day to Midsummer-Day, which is on the 24th of June, is reckoned the first quarter; from Midsummer-Day to Michaelmas-Day, which is on the 29th of September, is the second quarter; from Michaelmas-Day to Christmas-Day is the third quarter, and from Christmas-Day to Lady-Day is the last quarter of the year.* How many days in each quarter?

54. New York, Aug. 18, 1844.

Harcourt & Elliott, **Bought of Wm. Thompson,**

6 pieces of coarse cottons, each $32\frac{1}{2}$yd., at $10\frac{1}{2}$ct.
40 pieces " " each $31\frac{3}{4}$yd., at 11ct.
46 pieces fine " each $28\frac{1}{4}$yd., at $18\frac{7}{8}$ct.
49 pieces Merrimack prints, each 30yd., at $12\frac{1}{2}$ct.
28 pieces bleached muslins, each $29\frac{5}{8}$yd., at 15ct.
11 pieces Dover prints, each $81\frac{1}{2}$yd., at $18\frac{1}{2}$ct.

Ans. $695.48.

55. May-Day, on the 1st of May; Lammas-Day, on the 1st of August; Martinmas-Day, on the 11th of November; and Candlemas-Day, on the 2d of February, are sometimes kept as quarter-days in England.* How many days in each quarter?

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* Nesbit.

56. London, July 25, 1844.

Watson, Miller & Co., Bought of Joseph Merriam,

4 bags Surinam cotton,	each 3cwt. 1qr. 16lb.,	at 21¼d.
3 bags " "	each 2cwt. 3qr. 27lb.,	at 22¼d.
4 bags St. Domingo "	each 3cwt. 0qr. 10lb.,	at 14¾d.
4 bags Barbadoes "	each 2cwt. 1qr. 15lb.,	at 14¼d.
5 bags Pernambuco "	each 2cwt. 2qr. 19lb.,	at 21½d.

Ans. £510 3*s.* 3*qr.*

57. What is the yearly rent of 247½ acres of land, at $9.50 per acre? *Ans.* $2351.25.

58. What is the balance of the following account, the balance being the difference between the debtor and creditor amounts?

Dr.	*Cr.*
1169.42¾	432.50
875.18¾	6911.47⅛
9121.75	853.25
638.12½	419.37½
991.14⅞	1640.61¾

Ans. $2538.42¼.

59. How much wool, at 37½ct. a pound, will purchase 96 barrels of apples, at $1.66⅔ per barrel?

60. How many days are there in each of the four seasons; spring commencing on the 20th of March, when the sun enters Aries; summer on the 21st of June, when the sun enters Cancer; autumn on the 23d of September, when the sun enters Libra, and winter on the 21st of December, when the sun enters Capricornus?

61. The average amount of rain falling annually at Philadelphia, as determined by observations from 1810 to 1837, inclusive, is 36.99 inches.* At this average rate, how many cubic miles of rain would fall annually in the

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* American Almanac, 1839.

United States and territories, their area being 3252100 square miles?

Ans. 1898.598 *cubic miles, equivalent to a lake of the same area as the New England States,* (64230 *square miles,*) *and* 156.07 *feet deep.*

62. Find the value of the following obsolete English coins, in Federal Money, estimating the pound sterling at $4.84. A Moidore = £1 7s.; a Jacobus = £1 5s.; a Carolus = £1 8s.; a Mark = 18s. 4d.; an Angel = 10s.; a Noble = 6s. 8d.

63. What would be the value of $\frac{4}{13}$cwt. of gold, at the mint valuation, $18.84 per ounce Avoirdupois?

Ans. $10388.08.

64. What is the value of 8131 lb. of tea, at 62$\frac{1}{2}$ct. per lb.?

65. A merchant gained $1920 a year, for the first three years after commencing business; for the next four years he gained $2870.50, and for the next two he lost $1875.75 a year. How much had he gained at the expiration of the ninth year?

66. A pound Avoirdupois is equal to 7000 grains Troy. What is the value in Avoirdupois weight of 16lb. 5oz. 10dwt. 13gr. Troy? *Ans.* 13*lb.* 8*oz.* 11.4*dr*

67. Nine men bought 27 barrels of flour, at 5\frac{3}{7}$ per barrel. How much did each pay?

68. Determine in feet and inches the height of Goliath, which was six cubits and a span,* estimating the cubit at 20.8 inches, and the span at 10.4 inches.

69. How many gills in 16 casks, each holding 35$\frac{1}{7}$ gallons? *Ans.* 17993$\frac{1}{7}$*gal.*

70. A boy having some peaches, found, after giving

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* 1 Samuel, xvii. 4.

away 2 more than $\frac{1}{3}$, that 1 more than $\frac{1}{2}$ remained. How many had he at first?

[The difference between $\frac{1}{3}$ of the number and $\frac{1}{2}$ of the number, is 3.]

71. A boy being asked how many marbles he had, replied, "If I had as many more, and $\frac{1}{2}$ as many more, I should have 45." How many had he?

72. How many noggins in 5gal. 3qt. $1\frac{1}{2}$pt., a noggin being equivalent to a gill?

73. Add $16\frac{5}{9}$, $4\frac{2}{3}$, $17\frac{1}{6}$, $491\frac{3}{4}$, and $21\frac{7}{16}$.

Ans. $551\frac{83}{144}$.

74. $916.4 \times .0015 \div 363.18 = ?$ *Ans.* $.008\frac{4751}{6053}$.

75. Cape Horn is in 67° 16′ 8″ W. longitude, and 55° 58′ 40″ S. latitude; Cape of Good Hope is in 18° 28′ 45″ E. longitude, and 33° 56′ 3″ S. latitude.* Required the difference of latitude and of longitude between the two places.

76. $.0091 \times .00087 \div 1950000 = ?$

Ans. $.0000000000017\frac{4}{15}$.

77. Reduce to their lowest terms, $\frac{646}{238}$; $\frac{112}{532}$; $\frac{232}{343}$; $\frac{132}{627}$; $\frac{138}{828}$.

78. What is the value of a farm 189.5 rods long and 150 rods wide, at $37.50 per acre? *Ans.* $6662.11.

79. What is the value of £63.795, at $\frac{1}{2}$ cent per farthing?

80. What part of a solar year is 11 days 5 hours?

Ans. $\frac{96840}{3155693}$.

81. What part of a rod is 1ft. 7.8in.?

82. The average annual amount of rain falling at Providence, R. I., from 1832 to 1848, inclusive, was 38.41

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* United States Almanac.

inches.* At this rate, how many hogsheads would fall in the state of Rhode Island monthly, the area of the state being 1200 square miles? *Ans.* 1059551608 + *hhd.*

83. What part of a yard is 3qr. 1na. 1.5in.?

Ans. $\frac{41}{48}$.

84. What part of £4 11s. 3d. is £1 5s. $2\frac{3}{4}$d.?

Ans. $\frac{1211}{4380}$.

85. What part of 5℔ 7℥ 3ʒ is 2℔ 4℥ 1℈?

Ans. $\frac{673}{1617}$.

86. What part of 4gal. 3qt. is 2qt. 1pt. 3gi.?

87. Estimating the circumference of the earth's orbit at 596902654 miles, and the length of a year at 365dy. 5h. 48m. 50sec., how many miles per second does the earth average in its revolution round the sun? How many miles per hour? *Ans* 18.915*m. per sec.;* 68094.379*m. per h.*

88. What part of 4T. 3cwt. 1qr. is is $2\frac{3}{7}$T.?

Ans. $\frac{1360}{2331}$.

89. What part of $2\frac{5}{9}$cwt. is $18\frac{3}{5}$lb.? *Ans.* $\frac{837}{12880}$.

90. A cubic foot of distilled water weighs 1000 ounces Avoirdupois. What is the weight of a wine gallon?

Ans. 8*lb.* $5\frac{49}{72}$*oz.*

91. What will $\frac{2}{3}$ of $15\frac{3}{8}$ yards of broadcloth cost, at $\frac{3}{4}$ of $7.25 per yard? *Ans.* 55.73\frac{7}{16}$.

92. If A. can do a piece of work in 6 days, B. can do it in 9 days, and C. can do it in 12 days, in what time can it be done if they all work together? *Ans. In* $2\frac{10}{13}$*dy.*

93. What number is that, which, increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itself, makes 48? *Ans.* $23\frac{1}{25}$.

94. In a certain orchard $\frac{1}{2}$ the trees bear apples, $\frac{1}{4}$ bear peaches, $\frac{1}{6}$ bear cherries, and the remaining 30 bear pears.

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* American Almanac, 1850.

How many trees are there in the orchard, and how many of each sort? *Partial Ans.* 360 *in all.*

95. In the 4 years ending in 1784, in a badly-ventilated hospital in Dublin, 2944 children died out of 7650; but, after freer ventilation, the deaths in the same period of time, and in a like number of children, amounted only to 279.* At this rate, how many lives could be saved annually, by improved ventilation, in a population of 20000? *Ans.* 6967+.

96. What is the value of $\frac{5}{11}$ of a ton? *Ans.* 9*cwt.* 10*lb.* $2\frac{10}{11}$*oz.*

97. How many inches in $\frac{5}{16}$ of a mile?

98. Reduce 11oz. 7dwt. 3gr. to the fraction of a lb.

99. Reduce 4.79oz. to the fraction of a pound Avoirdupois. *Ans.* $\frac{479}{1600}$*lb.*

100. Reduce 38.756lb. to the fraction of a ton. *Ans.* $\frac{9689}{5600000}$.

101. Reduce 4.763in. to the fraction of a mile. *Ans.* $\frac{433}{5760000}$.

102. Reduce $\frac{3}{5}$ of a qr. to the decimal of a ton. *Ans.* .0075*T.*

103. Reduce $4\frac{7}{8}$ pence to the decimal of a pound. *Ans.* £.0203125.

104. Reduce 3.725 shillings to the fraction of a pound.

105. Reduce 59.63 minutes to the fraction of a day. *Ans.* $\frac{5963}{8640000}$.

106. Reduce $\frac{5}{9}$ of a day to hours, minutes, &c.

107. Mr. Edward J. Lance, of Blackwater, England, by sowing a single grain of wheat early in the season, and subdividing the roots and carefully cultivating them, ob-

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* British Sanitary Reports.

tained 43000 grains as a product.* At this rate, what would be the produce of $3\frac{2}{7}$ gills?

Ans. 551*bu.* 3*pk.* 5*qt. nearly.*

108. If 9 men can reap a field in $3\frac{1}{2}$ days, how long will it take 2 men to reap it? *Ans.* $15\frac{3}{4}$*dy.*

109. How much rice, at $4\frac{3}{4}$ cents a pound, can be bought for $1.50? *Ans.* $31\frac{11}{19}$*lb.*

110. If $29\frac{1}{3}$ gallons run from a cistern in an hour, how many pints will run out in $16\frac{5}{8}$ hours? *Ans.* $3901\frac{1}{3}$*pt.*

111. What is the price of $3\frac{3}{7}$cwt. of rice, at $4\frac{1}{2}$ct. per pound?

112. Estimating the British National Debt at 800 million pounds, and the average weight of 30 ten-pound notes at 1oz. Avoirdupois, what would be the weight of a number of ten-pound notes sufficient to pay the debt?

113. In the last 65 years, it is estimated, on the authority of official documents, that our military establishments have subjected us to an expense of at least $750000000. If such an amount should be invested so as to pay 6 per cent. interest, what annual income would it yield?

114. The number of depositors in savings-banks in Great Britain, on the 20th of November, 1848, was 1054663; the total amount deposited, $140230695.† What was the average to each depositor?

115. It is estimated that the aggregate yearly issue of newspapers, magazines, and reviews, from the city of New York alone, in the year 1849, was 72710257.† What would be their value, at the average rate of $2\frac{1}{2}$ cents per copy?

116. The Croton water-works of New York are capable

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* Patent Office Report, 1847.

† Family Christian Almanac, 1851.

of discharging 60000000 gallons every 24 hours. To what average amount per hour would this be equivalent?

117. London is supplied with water for drinking and culinary purposes, by eight different water-works, which it was estimated, in 1845, delivered 45 million gallons every 24 hours.* What was the average hourly consumption, and what was the total amount consumed in the year?

118. At $12\frac{1}{2}$ cents a yard, what will be the cost of 13yd. 3qr. of sheeting? *Ans.* \$1.71$\frac{7}{8}$.

119. If $\frac{7}{9}$ of a bushel of wheat cost \$0.75, what will be the price of $3\frac{3}{4}$ pecks? *Ans.* \$.90.

120. In $\frac{555}{20}$ of a pound, there are how many pounds? In 555 shillings, how many pounds?

121. In $\frac{8762}{24}$ of a day, there are how many days? In 8762 hours, how many days?

122. In $\frac{75968}{63}$ of a hogshead, there are how many hogsheads? In 75968 gallons, how many hogsheads?

XII. PERCENTAGE.

78. IT is customary to estimate all sums paid as commission or brokerage for the sale or purchase of property, insurance against loss by fire or otherwise, interest for the use of money, &c., at a given number of hundredths, called the *rate per cent.* (from the Latin *per centum*, signifying *by the hundred.*) Thus, 6 per cent. is .06; $5\frac{1}{4}$ per cent. is $.05\frac{1}{4}$ or .0525; $7\frac{1}{3}$ per cent. is $.07\frac{1}{3}$ or $.07\dot{3}$. To compute any percentage, *Multiply by the rate expressed decimally.*†

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* Family Christian Almanac, 1851.

† See Sect. 29, Ex. 47–71.

79. COMMISSION is an allowance of a certain percentage to a Factor, Correspondent, or Broker, for buying or selling property.

1. A broker sold goods at a commission of 4 per cent., for which he received $2500. What was the amount of his commission, and how much did he pay to his employer?

2. How much shall I receive for my real estate, by selling it for $10000, after allowing a commission of $2\frac{1}{2}$ per cent.? *Ans.* $9750.

3. My correspondent has expended $987.50 in the purchase of cloths, by my order. What will he receive for the transaction, if I allow him 2 per cent. on the whole amount?

4. What is the commission on £500 17s. 6d., at $2\frac{1}{8}$ per cent.? [Multiply by $\frac{17}{8}$ and divide by 100.]
Ans. £10 12*s.* $10\frac{1}{2}$*d.*

80. INSURANCE is a security against loss by fire or otherwise, and is generally obtained by the payment of a certain percentage, called the PREMIUM.

The instrument by which insurance is effected, is called the POLICY.

1. A farmer insured his house for $600, at $1\frac{1}{4}$ per cent.; his barn for $400, at $1\frac{1}{2}$ per cent., and his granary and sheds for $150, at 1 per cent. What premium did he pay for the whole? *Ans.* $15.

2. An English merchant effected an insurance of £2750 10s. on the ship Mary Ann, at a premium of $1\frac{1}{2}$ per cent. What did he pay, including a charge of 3s. 6d. for the policy? *Ans.* £41 8*s.* $7\frac{3}{4}$*d.*

3. What premium must be paid for insuring $8375 on a warehouse, at $1\frac{7}{8}$ per cent.?

4. What amount of premium must I pay to insure the following property for 1 year: \$3500.00 on my store, and \$7500.00 on the merchandise in the same, at $\frac{3}{4}$ per cent.; \$2750.00 on my house, and \$1750.00 on the furniture, books, and clothing contained in it, at $\frac{1}{2}$ per cent.; and \$600 on my barn, at $\frac{5}{8}$ per cent.?

81. TAXES are certain amounts levied on all citizens for the support of government. The poll-tax is assessed on every male citizen above a certain age specified by law, and its amount is fixed by statute. The property-tax is estimated at a certain percentage of its assessed value.

1. When taxes are rated at $\frac{3}{4}$ per cent. upon all real and personal estate, what amount will be paid by a man whose property is valued at \$9325, and who pays for 3 polls, at \$0.75? *Ans.* \$72$\frac{3}{16}$.

2. The poll-tax being \$1.50, and the property-tax \$4.50 on every \$1000, how much must I pay for \$10300 real estate, \$830 personal property, and 4 polls? *Ans.* \$56.08$\frac{1}{2}$.

3. At $\frac{3}{4}$ per cent., what amount of tax must be paid on property valued at \$27500? How much would be paid on the same property, at 1$\frac{1}{3}$ per cent.? At $\frac{2}{3}$ per cent.? At $\frac{3}{7}$ per cent.?

82. STOCKS are sums of money invested in government funds or corporations, and are bought and sold by *shares.* The *par value* of a share is the original cost. When a share will sell for more than the original cost, it is said to be *above par,* or *at an advance;* when for less, *below par,* or *at a discount.*

1. What must I pay for 49 shares of the Boston and Worcester Railroad, the stock being 12$\frac{1}{2}$ per cent. above par, and the par value \$100 per share? *Ans.* \$5512$\frac{1}{2}$.

2. The par value of United States Bank stock was \$100

per share. How much was received for 27 shares, at $6\frac{1}{4}$ per cent. below par? *Ans.* $2531.25.

3. A man purchased $4000 Pennsylvania stock, at par, and sold it at a discount of 12 per cent. How much did he receive for the whole?

4. What must I pay for 20 shares of bank stock, at an advance of $4\frac{3}{4}$ per cent. on the par value, which is $250 per share? *Ans.* $5237.50.

5. What is the value of $8400 of stock, at 25 per cent. advance? At $7\frac{2}{3}$ per cent. discount? At 50 per cent. below par?

83. GAIN AND LOSS are frequently estimated by percentage.

1. How much per cent. do I gain, by selling at 90ct. a pound, tea that cost me 75 cents? [The gain is 15ct., or $\frac{15}{75}$ of the prime cost. This fraction reduced to a decimal, gives .2, or .20.] *Ans.* 20 *per cent.*

2. 3.5 is what per cent. of 31? *Ans.* $11\frac{9}{31}$ *per cent.*

3. What per cent. of 28 is 7? 12? 13.7? 49.25?

4. Bought 36.75 yards of broadcloth, at $4.00 per yard, and sold the whole for $183.75. What did I gain, and how much per cent.? *Ans.* $36.75; 25 *per cent.*

5. A grocer sold sugar at $7\frac{1}{2}$ cents a pound, which was 25 per cent. more than it cost him. What was the prime cost? In other words, 0.075 is 1.25 times what number? *Ans.* 6 *cents per lb.*

6. 2.25 is .90 of what number? Then, if I lose 10 per cent. by selling broadcloth at $2.25 per yard, what was the prime cost?

7. A draper sold cloth at $12\frac{1}{2}$ per cent. less than it cost him, thereby losing $37\frac{1}{2}$ cents a yard. What did he pay for the cloth? *Ans.* $3 *a yard.*

8. When sugar is \$5.60 per cwt., what must be the price per lb. to gain 40 per cent.?

9. A merchant purchased 100 bushels of wheat, at \$0.95 per bushel, and paid \$1.25 for carting it from the farm. For how much must he sell one half of it to gain 10 per cent.? *Ans.* \$52.93¾.

10. Bought in Jamaica 20 hogsheads of molasses, each holding 118gal. 2qt., at 20 cents a gallon; paid for freight, \$2 per hogshead; for insurance, \$6.80; and for duties, \$48. At what price per gallon must I sell it to gain 25 per cent.? *Ans.* 80 *cents.*

11. Bought 12cwt. 3qr. 14lb. of coffee, at 9 cents a pound, but it having been damaged, I sold the whole for \$108.15. How much did I lose, and what per cent.?

2d Ans. 16⅔ *per cent.*

12. Bought 40 reams of paper for \$150. For how much must I sell it per ream, to gain 10 per cent.? What did it cost per sheet, there being 24 sheets in a quire, and 20 quires in a ream?

84. DUTIES are imposts levied by government on goods imported into a country. Before computing them, certain deductions are made. For leakage and breakage, 2 per cent. is allowed on the gauge on all merchandise paying duty by the gallon, contained in casks; 10 per cent. on all beer, ale, and porter, in bottles, and 5 per cent. on all other liquors in bottles.

DRAFT is an allowance made for waste. The following is the present legal rate:

On 1cwt. or 112lb.	1lb.
Above 1 — and under 2cwt.	2lb.
“ 2 — “ 3 —	3lb.
“ 3 — “ 10 —	4lb.
“ 10 — “ 18 —	7lb.
18 — and upwards,	9lb.

TARE is an allowance made for the weight of the box, cask, &c., containing the merchandise, and is made after deducting the draft. The weight, before making the deductions, is called the *gross weight;* after the deductions are made, the *net weight.* Duties are computed on the net weight.

1. What is the duty on 8cwt. 2qr. 27lb. of currants, tare 12 per cent., duty 3 cents a pound? *Ans.* $25.74.

2. What is the duty on 283lb. of chocolate, tare 10 per cent., duty 4 cents per pound? *Ans.* $10.08.

3. What is the duty on 927gal. 2qt. of wine, at 25ct. a gallon? *Ans.* $227.24.

4. What is the duty on $3\frac{7}{8}$cwt. of indigo, tare 12 per cent., duty 5ct. a pound? *Ans.* $18.92.

5. What is the duty on $4\frac{5}{7}$cwt. of pepper, tare 6 per cent., duty 5ct. a pound? *Ans.* $24.63.

85. INTEREST.

INTEREST is an allowance of a certain per cent. for the use of money. The *Principal* is the sum on which the interest is paid. The *Rate* is the percentage paid annually. It is regulated by statute, the usual rate, in the United States, being 6 per cent.* The *Amount* is the sum of principal and interest.

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* In each of the New England States, in New Jersey, Pennsylvania, Delaware, Maryland, Virginia, North Carolina, Tennessee, Kentucky, Ohio, Indiana, Illinois, Missouri, Arkansas, and the District of Columbia, and on United States notes, the rate is 6 per cent. In New York, South Carolina, Michigan, Wisconsin, and Iowa, it is 7 per cent. In Georgia, Alabama, Mississippi, Texas, and Florida, 8 per cent. In Louisiana, 5 per cent., though the bank interest is .06, and conventional interest may be as high as .10. In Maryland, the interest on tobacco contracts is .08. In Mississippi, Missouri, and Arkansas, the interest, by agreement, may be as high as .10; and in Illinois, Wisconsin, and

When interest is charged only on the original principal, it is called SIMPLE INTEREST. But when it is also charged on the interest, as it becomes due, it is called COMPOUND INTEREST.

86. SIMPLE INTEREST.

The usual rate is .06, and this rate is always to be understood, in the following pages, unless some other is expressed.

If the rate is .06 for one year, for one month it will be $\frac{1}{12}$ as much, or .005; for 6 days, or $\frac{1}{5}$ of 1 month, it will be $\frac{1}{5}$ as much as for one month, or .001. For 1 day, it will be $\frac{1}{6}$ as much as for 6 days, or $\frac{1}{6}$ of .001, which is $\frac{1}{6000}$. Hence the

BANK RULE.

Multiply the principal by the number of days, and divide by 6000*; the quotient will be the interest at* 6 *per cent.*

For any other rate than 6 *per cent., compute the interest in the same manner, and add or subtract such part as may be required. For* 4 *per cent., subtract* $\frac{1}{3}$*; for* $4\frac{1}{2}$ *per cent., subtract* $\frac{1}{4}$*; for* 5 *per cent., subtract* $\frac{1}{6}$*; for* 7 *per cent., add* $\frac{1}{6}$*, and so on.*

This rule is the one generally employed by banks and merchants. It estimates the year at 12 months of 30 days, and consequently gives a little excess of interest.

GENERAL RULE.

Multiply the time, expressed in years and parts of a year, by the annual rate, and multiply the principal by this product.*

Iowa, as high as .12. In California, no rate of interest has been established by law.

* Sometimes it is most convenient to express the parts decimally; at others, fractionally.

87. EXAMPLES FOR THE BOARD.

1. Find the bank interest of $1963.50, from January 29th to May 2d.

Jan. has 31 days.
Feb. " 28 "
March " 31 "
April " 30 "
May " 2 "
122
29
93

If we should continue counting all the intervening days, as if they belonged to January, May 2d would correspond to January 122d. 29 from 122 leaves 93, the number of days the principal is at interest. 1963.50 × 93 ÷ 6000 = $30.43.

2. Find the exact interest of $4725, for 5mo. 19dy.

5mo. 19dy. = $\frac{169}{30}$mo. = $\frac{169}{360}$yr. $\frac{169}{360} \times \frac{6}{100} = \frac{169}{6000}$. $4725 × $\frac{169}{6000}$ = $133.08.

88. EXAMPLES FOR THE PUPIL.

1. What is the bank interest on $950, for 6 months, (183 days,) at 6 per cent.? *Ans.* $28.975.

2. What is the interest on $1375, for 11mo. 5dy., ($\frac{67}{72}$yr.,) at 5 per cent.? *Ans.* $63.98.

3. What is the exact interest on $2000, for 475 days, ($1\frac{22}{73}$yr.,) at 4 per cent.? *Ans.* $104.11.

4. What is the exact interest on $1500, from August 18th, 1843, to September 7th, 1844, at 6 per cent.? *Ans.* $95

5. What is the bank interest on $942.50, from February 11th, 1844, to December 3d, 1844, at 6 per cent.?

6. What is the bank interest on $23635, for 63 days, at $4\frac{1}{2}$ per cent.?

7. What is the bank interest on $75.10, for 93 days, at 7 per cent.?

8. What is the interest on $4000, from June 1st, 1841, to May 13th, 1844, at 6 per cent.? *Ans.* $708.

9. What is the interest on $181.25, for 2yr. 4mo. 19dy., at 5 per cent.? *Ans.* $21.62.

10. What is the interest on $95, for 7yr. 3mo., at 6 per cent.?

11. What is the amount of $5431, for 33 days, at 6 per cent., bank interest?

12. What is the bank interest of $47965, for 34 days, at 5 per cent.?

13. What is the amount of $10000, for 64 days, at 7 per cent.?

14. What is the interest of $5387.75, from 10th mo. 3d, 1842, to 9th mo. 1st, 1844, at 4½ per cent.? *Ans.* $464.02.

15. What is the amount of $99.50, from July 4th, 1839, to June 27th, 1843, at 6 per cent.? *Ans.* $123.26.

16. What is the bank interest of $500.00, for 123 days, at 5 per cent.?

17. What is the bank interest of $2500, for 94 days, at 4 per cent.? What is the amount?

18. What is the amount of $730.00, from December 1st, 1829, to January 31st, 1835, at 7 per cent.? *Ans.* $994.02.

19. What is the amount of $629.50, from 8th mo. 3d, 1840, to 7th mo. 2d, 1842? *Ans.* $701.79.

20. Bought 75¾ pounds of tea, at 62½ cents per pound, for which I gave my note, payable in 6 months, dated June 5th, 1848. What was the amount of the note, March 27th, 1849? [Interest does not commence until the note becomes due.] *Ans.* $48.23.

21. John Smith gave a note for $2000 to the Bank of North America, payable in 60 days. How much money did he receive, after deducting 63 days' interest?

22. How much should I receive on a note at 90 days, for $639.50, after deducting 93 days' interest?

23. What is the present worth of a 30 days' note for $1870, after deducting 33 days' interest?

24. What is the amount of a note for $2131.00, for 183 days, at 6 per cent., bank interest?

89. PRACTICE RULE.

Multiply 1 *per cent. of the principal by* $\frac{1}{2}$ *the even number of months, and if there is an odd month, add* 30 *to the number of days. Divide the days by* 6, *and multiply* $\frac{1}{10}$ *of* 1 *per cent. by the quotient. If there are any remaining days,* take as many* 60*ths of* 1 *per cent. Add the numbers so obtained, and their sum will be the interest at* 6 *per cent. For any other rate, increase or diminish the result, as in the Bank Rule.*

90. EXAMPLE FOR THE BOARD.

What is the interest of $9763.25 for 2yr. 9mo. 8dy.?

1 per cent. is	97.6325
$\frac{1}{2}$ of 2yr. 8mo. is	16
	585.7950
	976.325
36dy. is .6 of 2mo.	58.5795
2dy. is $\frac{1}{30}$ of 2mo.	3.2544
	$1623.9539

1 per cent., or the interest for 2 months, is $97.6325. Multiplying by 16, we obtain the interest for 2yr. 8mo. The 1mo. remaining, added to the 8 days, gives 38 days, in which 6 is contained 6 times, with 2 remainder. As the interest for 6 days is .1 of 1 per cent., or $9.763+, the interest for 36 days is 6 times as much. The 2 remaining days are $\frac{1}{30}$ of 60 days; we therefore add $\frac{1}{30}$ of $9.763+.

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* When the remainder is 4, it is more convenient to diminish the quotient by 1, and call the remainder 10, taking $\frac{10}{60}$ or $\frac{1}{6}$ of 1 per cent. for the interest.

91. EXAMPLES FOR THE PUPIL.

1. What is the interest of $281.50, for 3yr. 7mo. 16dy.? What is the amount? *Ans.* $61.27; $342.77.

2. What is the interest of $1175, for 1yr. 11mo. 29dy.? What is the amount? *Ans.* $140.80; $1315.80.

3. What is the interest of $28.75, for 2yr. 3mo. 4dy., at 5 per cent.?* What is the amount? *Ans.* $3.25; $32.

4. What is the interest of $3360, for 3yr. 3mo. 3dy., at 7 per cent.? What is the amount?

5. What is the amount of $20000, from September 30th, 1839, to March 27th, 1844, at 5 per cent.?

6. What is the amount of $531.50, from November 3d, 1840, to February 25th, 1842, at 7 per cent.?

7. What is the amount of $74.23, from June 29th, 1842, to September 13th, 1844, at $4\frac{1}{2}$ per cent.? *Ans.* $81.60.

8. Sold October 11th, 1830, 162bu. 3pk. of oats, at $37\frac{1}{2}$ cents a bushel, and received in payment a note payable in 3 months. What was the amount of the note, May 5th, 1840? *Ans.* $95.15.

92. INTEREST ON ENGLISH MONEY.

The readiest mode of computing percentage on English money, is to take .01 of each denomination, and multiply by the number of per cent. required. There is then but one reduction necessary.

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* First find the interest at 6 per cent., and take $\frac{5}{6}$ of it for the interest at 5 per cent.

93. EXAMPLES FOR THE BOARD.

1. What is 5 per cent. of £2845 13s. 11d.?

```
£28.45   .13s.  .11d.
                  5
---------------------
142.25   .65    .55
    20
  ----
  5.00
   .65
  ----
  5.65
    12
  ----
  7.80
   .55
  ----
  8.35
```

1 per cent. of £2845 is £28.45; .01 of 13s. is .13s.; .01 of 11d. is .11d. 5 per cent. is £142.25 .65s. .55d. Multiplying £.25 by 20, we have 5.00s., and adding the .65s., we get 5.65s. .65s. = 7.80d., to which adding the .55d., we have 8.35d. The answer is therefore £142 5s. 8.35d.

2. What is the interest on £128 3s. 4d., for 27 days?

```
   1.28    .03    .04
                  27
-------------------------  s.  d.  qr.
73)34.56    .81   1.08(9   5   3
      20
  ------
  692.01
  657
  ------
   35.01
      12
  ------
  421.20
  365
  ------
   56.20
       4
  ------
  224.80
  219
  ------
```

The regular rate in England is 5 per cent., and this rate is always understood in mercantile transactions, unless some other rate is stipulated. If the rate is .05 for 365 days, it is .01 for 73 days, and for 1 day it is $\frac{1}{73}$ of 1 per cent. Then, to find the interest for any number of days, we need only multiply .01 of the principal by the number of days, and divide by 73.

The custom is very general in England, of leaving the shillings and pence out of consideration, in the computation of interest. In this case, if the number of shillings in the principal is 10 or more, it is considered as another pound. Thus, in finding the interest on £27 5s. 9d., the principal would be regarded as £27; if the principal was £18 10s., it would be regarded as £19. But in the examples of the following section, the exact interest is required.

94. EXAMPLES FOR THE PUPIL.

1. What is 2 per cent. of £95 9s.?*
Ans. £1 18*s.* 2.16*d.*

2. What is 10 per cent. of £18 3s. 6d.?
Ans. £1 16*s.* 4.2*d.*

3. What is $7\frac{1}{2}$ per cent. of £40 2s. 11d.?
Ans. £3 $2\frac{5}{8}$*d.*

4. What is 25 per cent. of £4 19s.?

5. What is the interest of £17 5s., for 19 days?
Ans. 10*d.* 3*qr.*

6. " " £3 10s. 6d., for 20 days?
Ans. 2*d.* 1*qr.*

7. " " £4 4s., for 1mo. 15dy.?
Ans. 6*d.* 1*qr.*

8. " " £2 8s. 9d., for 3mo. 3dy.?
9. " " £44 2s. 2d., for 15dy.?
10. " " £200, for 123 days?
11. " " £68 7s. 7d., for 10 days?
12. " " £135 14s., for 68 days?
13. " " £27 11s., for 39 days?

95. PARTIAL PAYMENTS.

When partial payments are made on mercantile accounts which are past due, it is customary to compute interest on the whole debt from the time it became due, and on each payment from the time it was made until the time of settlement, and to deduct the amount of all the payments, including interest, from the amount of the debt and interest.

The labor of computing interest on each item may be avoided, by multiplying the amount due at first, and the balance due after each payment, by the number of days they are severally at interest, adding all the products, and dividing the amount by 6000. The quotient will be the interest at 6 per cent.*

---

* For other methods of computing interest on English money, see "Arithmetic Practically Applied," p. 214.

96. EXAMPLE FOR THE BOARD.

A debt of $630.25 became due March 15, on which the following payments were made: April 3, $170; May 20, $245.30; June 17, $87.50. How much was due, Sept. 5, when the account was settled?

	$	Days.	Products.
March 15, Amount due . . .	630.25	× 19 =	11974.75
April 3, first payment . . .	170.		
Balance	460.25	× 47 =	21631.75
May 20, 2d payment . . .	245.30		
Balance	214.95	× 28 =	6018.60
June 17, 3d payment . . .	87.50		
Balance	127.45	× 80 =	10196.00
			6000)49821.10
			8.30351

Ans. $127.45 + $8.30 = $135.75.

The whole debt is on interest from March 15 to April 3, 19 days. The principal is then reduced, by a payment, to $460.25, which is on interest from April 3 to May 20, 47 days. In like manner we find that the 2d balance is on interest 28 days, and the 3d, 80 days. The amount due at settlement, is the unpaid balance of the debt, —$127.45, together with the interest, $8.30.

97. EXAMPLES FOR THE PUPIL.

1. Sold goods to the value of $375, which should have

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* It may be readily seen that the same result will be obtained, whichever method is adopted; but no accountant who is familiar with both methods, will hesitate to adopt the latter.

been paid August 18, 1850. Required the amount due January 1, 1851, the following payments having been made on account: Aug. 31, $100; Oct. 1, $100; Nov. 15, $150.
Ans. $28.74.

2. On an account of $428.50, due April 4, 1849, the following payments were made: May 5, $125; July 20, $150; Sept. 1, $100. How much was due Oct. 15?
Ans. $61.05.

3. The amount of an account due Jan. 1, 1850, was $580.75. What was the balance due July 1, computing interest at 7 per cent., the following payments having been made on account?* Feb. 13, $137.25; April 8, $120; May 3, $203.50; June 10, $68.75. *Ans.* $63.43.

4. Find the balance due July 28, 1850, on an account of $813.23, due Jan. 1, 1850, on which are the following payments: March 1, $200; April 3, $250; May 5, $75.23; June 2, $218.50; June 27, $29.65. Interest at 8 per cent. *Ans.* $60.04.

98. COMPOUND INTEREST.

It is sometimes stipulated that the interest shall be paid quarterly, semi-annually, or annually; and if not paid at the specified time, the interest is added to the principal, and the amount is regarded as a new principal, on which interest is computed to the time it again becomes due. Such interest is called COMPOUND INTEREST, but, though often charged, it is not strictly legal.

To find the amount of any sum at compound interest, *Increase the principal by the interest each time it becomes due, and consider the amount a new principal.*

---

* The same method may be pursued as in each of the preceding examples, increasing the interest obtained at the end of the operation by $\frac{1}{6}$ of itself.

99. EXAMPLE FOR THE BOARD.

What will be the amount of $280, for 1yr. 10mo. 22dy., interest payable semi-annually?

280
.03 rate for 6mo.

8.40
280

288.40 amt. for 6mo.
.03

8.6520
288.40

297.052 amt. for 1yr.
.03

8.91156
297.052

305.96356 amt. for 1yr. 6mo.
.0236 rate for 4mo. 22dy.

7.220740016
305.96356

313.1843 amt. for 1yr. 10mo. 22dy.

The interest may be obtained by subtracting the principal from the amount. In the present example, the interest is $313.18 — $280.00 = $33.18.

100. EXAMPLES FOR THE PUPIL.

1. What is the amount of $150, for 2 years, at 6 per cent. compound interest? *Ans.* $168.54.

2. What is the amount of $225, for 3 years, at 5 per cent., compound interest? *Ans.* $260.46.

3. What is the amount of $500 for 4 years, at 7 per cent., compound interest?

4. What is the compound interest on $437.50, for 2 years, interest payable semi-annually? *Ans.* $54.91.

5. Find the compound interest on $300, for 3yr. 8mo., at 8 per cent.? *Ans.* $98.07

TABLE

SHOWING THE AMOUNT OF $1.00, AT COMPOUND INTEREST, FROM 1 YEAR TO 50.

Year.	3 p. cent.	3½ p. cent.	4 p. cent.	4½ p. cent.	5 p. cent.	6 p. cent.	7 p. cent.
1	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.229874	1.272279	1.315932	1.360862	1.407100	1.503630	1.605781
8	1.266770	1.316809	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.304773	1.362897	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.343916	1.410599	1.480244	1.552969	1.628895	1.790848	1.967151
11	1.384234	1.459970	1.539454	1.622853	1.710339	1.898299	2.104852
12	1.425761	1.511069	1.601032	1.695881	1.795856	2.012196	2.252192
13	1.468534	1.563956	1.665073	1.772196	1.885649	2.132928	2.409845
14	1.512590	1.618694	1.731676	1.851945	1.979932	2.260904	2.578534
15	1.557967	1.675349	1.800943	1.935282	2.078928	2.396558	2.759031
16	1.604706	1.733986	1.872981	2.022370	2.182875	2.540352	2.952164
17	1.652848	1.794675	1 947900	2.113377	2.292018	2.692773	3.158815
18	1.702433	1.857489	2.025816	2.208479	2.406619	2.854339	3.379931
19	1.753506	1.922501	2.106849	2.307860	2.526950	3.025599	3.616526
20	1.806111	1.989789	2.191123	2.411714	2.653298	3.207135	3.869683
21	1.860295	2.059431	2.278768	2.520241	2.785963	3.399564	4.140561
22	1.916103	2.131512	2.369919	2.633652	2.925261	3.603537	4.430400
23	1.973586	2.206114	2.464715	2.752166	3.071524	3.819750	4.740528
24	2.032794	2.283328	2.563304	2.876014	3.225100	4.048935	5.072365
25	2.093778	2.363245	2.665836	3.005434	3.386355	4.291871	5.427431
26	2.156591	2.445959	2.772470	3.140679	3.555673	4.549383	5.807351
27	2.221289	2.531567	2.883369	3.282009	3.733456	4.822346	6.213866
28	2.287928	2.620177	2.998703	3.429700	3.920129	5.111087	6.648836
29	2.356565	2.711878	3.118651	3.584036	4.116136	5.418388	7.114255
30	2.427262	2.806794	3.243397	3.745318	4.321942	5.743491	7.612253
31	2.500080	2.905031	3.373133	3.913857	4.538039	6.088101	8.145110
32	2.575083	3.006708	3.508059	4.089981	4.764941	6.453387	8.715268
33	2.652335	3.111942	3.648381	4.274030	5.003188	6.840590	9.325337
34	2.731905	3.220860	3.794316	4.466361	5.253348	7.251025	9.978110
35	2.813862	3.333590	3.946089	4.667348	5.516015	7.686087	10.676578
36	2.898278	3.450266	4.103932	4.877378	5.791816	8.147252	11.423939
37	2.985227	3.571025	4.268090	5.096860	6.081407	8.636087	12.223614
38	3.074783	3.696011	4.438813	5.326219	6.385477	9.154252	13.079277
39	3.167027	3.825372	4.616366	5.565899	6.704751	9.703507	13.994827
40	3.262038	3.959260	4.801021	5.816364	7.039989	10.285718	14.974465
41	3.359899	4.097834	4.993061	6.078101	7.391988	10.902861	16.022677
42	3.460696	4.241258	5.192784	6.351615	7.761587	11.557033	17.144265
43	3.564517	4.389702	5.400495	6.637438	8.149667	12.250455	18.344363
44	3.671452	4.543342	5.616515	6.936123	8.557150	12.985482	19.628469
45	3.781596	4.702358	5.841176	7.248248	8.985008	13.764611	21.002461
46	3.895044	4.866941	6.074823	7.574420	9.434258	14.590487	22.472634
47	4.011895	5.037284	6.317816	7.915268	9.905971	15.465917	24.045718
48	4.132252	5.213589	6.570528	8.271455	10.401267	16.393872	25.728918
49	4.256219	5.396065	6.833349	8.643671	10.921333	17.377504	27.529043
50	4.383906	5.584927	7.106683	9.032636	11.467400	18.420154	29.457039

102. EXAMPLES TO BE SOLVED BY THE TABLE.

1. What is the amount of $500, for 19 years, at 5 per cent., compound interest? *Ans.* $1263.47.

2. What is the amount of $1000, for 50 years, at 7 per cent., compound interest?

3. What is the compound interest on $750, for 12 years, at 6 per cent.? *Ans.* $759.15.

4. What is the compound interest on $400, for 15 years, at 5 per cent.? *Ans.* $431.57.

5. What is the amount of $294, for 40 years, at 4 per cent., compound interest?

6. What is the amount of $600, for 37 years, at 3½ per cent., compound interest?

7. What is the amount of $800, for 25 years, at 4½ per cent., compound interest?

8. What is the amount of $587.50, for 18 years, at 3 per cent., compound interest?

103. PROBLEMS IN INTEREST.*

I. THE PRINCIPAL, INTEREST, AND TIME, BEING GIVEN, TO FIND THE RATE,—

Divide the given interest by the interest at 1 *per cent. for the given time.*

1. If I pay $68.705 for the use of $1963.00 six months, what is the rate per cent.? *Ans.* .07.

2. At what rate per cent. will $421.50 amount to $674.40 in 6yr. 8mo.? [The interest is found by subtracting the principal from the amount.]

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* Let the pupils exercise their ingenuity in explaining the rationale of these rules.

3. At what rate per cent. will a merchant double his capital in six years?

4. If a capital of $913 amounts to $1250 in 15 months, what is the annual gain per cent.? *Ans.* $.29\frac{483}{913}$.

II. THE PRINCIPAL, INTEREST, AND RATE, BEING GIVEN, TO FIND THE TIME,—

Divide the given interest by the interest for 1 *year, at the given rate.*

5. In what time will $1500, at 7 per cent., amount to $1920? *Ans.* 4 *years.*

6. The interest on a note of $750 was $131.25, and the rate was .05. What was the time?

7. A note of $431.00 amounted, at its settlement, to $546.29¼. How long had it been on interest, the rate being 6 per cent.? *Ans.* 4*yr.* 5*mo.* 15*dy.*

8. In what time will any principal be doubled, at 4 per cent.? At 5 per cent.? 6 per cent.? 7 per cent.?

III. THE TIME, RATE, AND INTEREST, BEING GIVEN, TO FIND THE PRINCIPAL,—

Divide the given interest by the interest of $1.00 *for the given time.*

9. What principal, at 6 per cent., will yield $150 in 18 months? *Ans.* $1666⅔.

10. A man's property yields 5½ per cent. interest. What is he worth, his annual income being $1375?

11. The quarterly expenditure of a charity-school is $600. What sum of money, yielding 8 per cent. per annum, will support it?

12. What is the face of a note which, at 7 per cent., will yield $111.65 interest in 3yr. 8mo.?

IV. THE TIME, RATE, AND AMOUNT, BEING GIVEN, TO FIND THE PRINCIPAL,—

Divide the given amount by the amount of $1.00 *for the given time.*

13. What principal will amount to $500.00 in 2yr. 6mo., —or what is the *present worth* of $500.00, due in 2yr. 6mo.? *Ans.* $434.78.

14. What is the present worth of $1844, due in 1yr. 8mo.? *Ans.* $1676.36.

15. What is the present worth of $725.50, due in 6yr., interest at 5 per cent.? *Ans.* $558.08.

16. What is the present worth of $2000.00, due in 3yr. 6mo., interest at 7 per cent.? *Ans.* $1606.43.

104. DISCOUNT.

DISCOUNT is an allowance for the payment of money before it is due. Equitable Discount is found by subtracting the present worth, (which is found by Problem IV. of the last section,) from the amount.

BANK, or BUSINESS DISCOUNT, is the same as Bank Interest. In computing discount, it is customary to add 3 days to the time stated in the note. These 3 days are called *days of grace*, and the note is not legally due until the last day of grace.

1. A note for $2000, payable in 30 days, was discounted at a bank. What was received for the note?
Ans. $1989.

2. What is the bank discount on a note for $1750, payable in 60 days? *Ans.* $18.375.

3. How much can I obtain by discounting the following note, at 7 per cent.?

$375.50 Albany, Sept. 19, 1850.

Ninety days from date, I promise to pay Jacob Murray, or order, three hundred and seventy-five dollars and fifty cents, for value received. MORRIS JOHNSON.

4. What are the present worth and the equitable discount of $1870, due in 3yr. 4mo. 13dy.?

Ans. Present worth, $1555.52; *Discount*, $314.48.

5. Wilmington, 5th mo. 7th, 1850.

For value received, I promise to pay Jonathan Mason, or order, six hundred and seventy-three dollars and twenty-five cents, in six months. MOSES A. WILSON.

$673.25

What ought Moses A. Wilson to pay, in order to cancel the above note, 8th mo. 3d, 1850? *Ans.* $662.87.

6. The Farmers' & Mechanics' Bank offer to discount the following notes at 5 per cent. How much shall I receive on each?

$200.00 Philadelphia, 8th mo. 3d, 1850.

Sixty days from date, I promise to pay Edward Morris, or order, two hundred dollars, for value received.

WILLIAM JACOBSON.

$144.00 Baltimore, Aug. 3d, 1850.

Ninety days from date, I promise to pay Edward Morris, or order, one hundred and forty-four dollars, for value received. GEORGE M. WATSON.

$75.75 New York, Sept. 5, 1850.

Six months from date, I promise to pay Edward Morris, or order, seventy-five dollars and seventy-five cents, for value received. MARCUS GRAHAM.

7. What is the bank discount on a note of $1270 for 30 days? 60 days? 120 days?

8. What is the difference between the equitable and the bank discount on $2000 for 1 year? *Ans.* $7.79.

9. How much do I gain by purchasing molasses for $1600 cash, and selling it immediately for $2000, payable in 9 months,—the People's Bank discounting the note at 7 per cent.? *Ans.* $293.83.

10. Sold George Manning & Brothers, 6¾cwt. of sugar, at 5½ct. a pound; 81 barrels of flour, at $4.62½ per barrel; 3.875cwt. of coffee, at 8⅝ct. a pound; 2cwt. 1qr. 14lb. of rice, at $3.50 per cwt.; 12 boxes sperm candles, each containing 32½lb., at 29ct. per lb.; and 19lb. chocolate, at 18¾ct. a pound. How much was received on their note for the amount, at 90 days, discounted at the National Bank?

11. Sold Clark & Atkins, 8cwt. 2qr. of sugar, at $5.00 per cwt.; 390lb. Young Hyson tea, at 87½ct. a pound; 250lb. Imperial tea, at 62½ct. a pound; 78gal. of molasses, at 31¼ct. a gallon; and 127lb. cheese, at 8¾ct. a pound, receiving their note at 120 days for the amount. What is the value of the note, discounting at 7 per cent.?

12. What is the value of a note at 60 days, for the following amounts? 163⅝yd. of silk, at 87½ct. a yard; 491½yd. India muslins, at 62½ct. a yard; 363yd. Alpaca, at 93¾ct. a yard; 224yd. merino, at 50ct. a yard; and 250yd. printed lawn, at 37½ct. a yard.

13. What is the value of a note at 30 days, for $24491.75, discounting at 5½ per cent.?

14. What is the value of a note at 60 days, for $1874.50, discounting at 4¾ per cent.?

15. What is the value of a note at 90 days, for $4959 375, discounting at 4½ per cent.?

XIII. PROPORTION, AND THE RULE OF THREE.

105. DEFINITIONS.

The RATIO of two numbers is the quotient resulting from the division of the first by the second. Thus, the ratio of 1 to 2, is $\frac{1}{2}$; of 13 to 6, $\frac{13}{6}$.

A ratio is usually expressed by two points written between the numbers, as 1 : 2; 13 : 6;* which are read 1 *is to* 2, or 1 *divided by* 2; 13 *is to* 6, or 13 *divided by* 6.

A PROPORTION is the union of two equal ratios, by writing four points, or the sign of equality, between them. Thus, 2 : 3 : : 4 : 6, or 2 : 3 = 4 : 6, is a Proportion, and may be read, 2 *is to* 3 *as* 4 *is to* 6, or 2 *divided by* 3 *equals* 4 *divided by* 6. The numbers themselves are called PROPORTIONALS.

106. PROPERTIES OF PROPORTIONS.

The first term of every ratio is called the *antecedent*, and the second, the *consequent*. The first and fourth terms are called the *extremes;* the second and third terms, the *means;* and in every proportion, *the product of the extremes is equal to the product of the means.* Take for example the proportion 3 : 9 : : 5 : 15. This may also be written $\frac{3}{9} = \frac{5}{15}$. Reducing these fractions to a common denominator, we have $\frac{45}{135} = \frac{45}{135}$. The numerators are now the same, but one is the product of the extremes, and the other the product of the means.

The antecedents and consequents may, therefore, change places in a variety of ways, the proportion always continuing so long as the product of the means is equal to the product of the extremes.

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* The sign of ratio (:) is merely the sign of division, (÷,) without the horizontal line.

Then, whenever one of the extremes and the two means are given, to find the other extreme, *Divide the product of the means by the given extreme.*

If any number will divide either antecedent and its consequent, the division may be performed without destroying the proportion. Thus, the proportion 9 : 12 : : 15 : 20 is the same as $\frac{9}{12} = \frac{15}{20}$. Dividing 9 and 12 by 3, or 15 and 20 by 5, the corresponding fraction will be reduced to its lowest terms, and the result, 3 : 4 : : 15 : 20, or 9 : 12 : : 3 : 4, is a proportion.

There are many other interesting properties of proportions, which belong more properly to the province of Geometry. What we have already stated, is sufficient for all Arithmetical purposes.

107. Proportional Analysis.

There is a large class of questions, in which the answer sought bears a determinate ratio to a known number. Such questions may be readily stated in the form of a proportion.

For example, if a certain number of men can perform a piece of work in a given time, twice as many men could perform twice as much work in the same time; one-half as many men could perform one-half as much; seven-eighths as many men could perform seven-eighths as much; and, generally, the amount of work performed, is proportioned to the number of men employed.

Thus, if 7 men can build 5 rods of wall in a day, how much could 21 men build in the same time? As we have just observed, 7 men must bear the same proportion to

21 men, as the work that seven men can do, to the work that 21 men can do. Stating this fact in the form of a proportion, we have

$$\overset{\text{men.}}{7} : \overset{\text{men.}}{21} :: \overset{\text{rods.}}{5} : \textit{Ans.}$$

Three terms of the proportion are given, and the fourth is required, which may be found by the method given in § 106. The means are 21 and 5, and the given extreme is 7. The required extreme is therefore $5 \times 21 \div 7 = 15$

108. INVERSE PROPORTION.

Sometimes the increase of a quantity gives a diminished result, and the decrease of the same quantity gives an increased result,—the diminution or increase being exactly proportioned to the increase or decrease of the original quantity.

Thus, if 6 men could do a piece of work in 12 days, then, to perform the work in one-half the time, twice as many men would be required; to perform it in twice the time, only one-half as many would be required; to perform it in $\frac{1}{3}$ of the time, three times as many would be required. In such examples, the proportion is said to be inverse.

If 6 men can do a piece of work in 12 days, in how many days could 3 men do it?

It would take 1 man 6 times as long as 6 men, and it would take 3 men only $\frac{1}{3}$ as long as 1 man, or $\frac{6}{3}$ as long as 6 men. The answer must therefore be $\frac{6}{3}$ of 12 days. To state this fact in the form of a proportion, we should have

$$\overset{\text{men.}}{3} : \overset{\text{men.}}{6} :: \overset{\text{days.}}{12} : \textit{Ans.}$$

Dividing the product of the means by the given extreme, we have $12 \times 6 \div 3 = 24$, which is the required extreme.

109. Statement of Questions by Proportion.

In all proportions, if the 4th term is greater than the 3d, the second term must also be greater than the 1st; if the 4th term is equal to the 3d, the second term must be equal to the 1st; if the 4th term is less than the 3d, the 2d term must be less than the 1st. This will be evident, if the ratios are reduced to fractions, and the sign of equality placed between them.

If, therefore, we perceive that a proportional relation exists between the three terms of a given question, whether the proportion be direct or inverse, the question may be stated by *writing the term which is of the same kind as the answer as the third term of a proportion, and considering whether the answer will be greater or less than the third term. If the answer will be greater, the greater of the two remaining quantities should be written as the second term, (and the less as the first term;) if the answer will be less, the less quantity should be the second term, (and the greater the first term.)*

110. EXAMPLE FOR THE BOARD.*

If 8 men build 40 rods of wall in 3 days, how many men will build 100 rods in 12 days?

To build 100 rods it will evidently take $\frac{100}{40}$ as many men as to build 40 rods, or $\frac{100}{40}$ of 8, which may be expressed by the proportion

$$\begin{array}{ccccccc} \text{rods.} & & \text{rods.} & & \text{men.} & & \text{men.} \\ 40 & : & 100 & :: & 8 & : & — \end{array}$$

To build the wall in 12 days, it will require but $\frac{3}{12}$ as many men as to build it in 3 days, which may be expressed by the proportion

$$\begin{array}{ccccccc} \text{days.} & & \text{days.} & & \text{men.} & & \text{men.} \\ 12 & : & 3 & :: & \frac{100}{40} \text{ of } 8 & : & — \end{array}$$

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* In explaining this example, the teacher should develope all the principles of the three preceding sections.

These two proportions may be written together, thus,

$$\begin{array}{l}\left.\begin{array}{r}40 : 100\\ 12 : 8\end{array}\right\} :: \overset{\text{men.}}{8} : \overset{\text{men.}}{—}\\ \hline 480 : 800 \;::\; 8 : —\end{array}$$

or by cancelling the like factors in the antecedents and consequents,

$$\begin{array}{l}\left.\begin{array}{r}2 : 5\\ 4 : 1\end{array}\right\} :: 8 : —\\ \hline 8 : 5 \;::\; 8 : —\end{array}$$

Multiplying together the two antecedents, and the two consequents of the first ratio, we obtain a single proportion, in which the two means and one extreme are given. Dividing the product of the means by the given extreme, we obtain 5 men for the answer, and our completed proportion is,

$$8 : 5 :: \overset{\text{men.}}{8} : \overset{\text{men.}}{5}\ \textit{Ans.}$$

Hence we derive the RULE OF THREE.*

Write that number which is of the same kind as the answer, for the third term of a proportion. Of the remaining quantities, compare any two of a kind, and consider whether the answer will be greater or less than the third term. If greater, write the greater number for the second term, and the less for the first. If less, write the less number for the second term, and the greater for the first. In the same way, compare any other two of a kind, and so proceed until all the numbers are employed. Cancel the like factors in the antecedents and consequents, and, to obtain the fourth term, or answer, multiply the third term by the product of the second terms, and divide by the product of the first terms.

N. B. Before cancelling, all compound numbers must be reduced to the same denomination.

The pupil should be required, after performing the examples in the following section, according to the rule, to give an analytical solution of each question.

111. EXAMPLES FOR THE PUPIL.

1. A cubic foot of distilled water weighs 1000 ounces.

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* The old RULE OF THREE embraced only questions in simple proportion, three terms being given, and a fourth required. When the result depended on five or more terms, the question was said to be in compound proportion.

What is the weight of a wine gallon, or 231 cubic inches? *Ans.* 133$\frac{49}{72}$*oz.*

2. If 4 yards of cassimere cost $10, what will 15 yards cost?

3. If a cistern discharges 45 gallons of water in 2 hours, how much will it discharge in 8 hours?

4. If the interest on a certain sum of money is $1750 for 3 years, what would be the interest on the same sum for seven years? *Ans.* 4083\frac{1}{3}$.

5. How far would a locomotive run in 13 hours, at the rate of 17 miles per hour?

6. What must I pay my clerk for 89 days' work, his wages being $9 per week?

7. How many men will build a wall in 17.5 days, that 8 men can build in 105 days? *Ans.* 48 *men.*

8. A family of 7 men use a barrel of flour in 74 days. How many men would use a barrel in 18.5 days?

9. The provisions of a garrison are sufficient to supply 736 men 14.5 days. How long will they serve 368 men? *Ans.* 29 *days.*

10. How many yards of serge, that is $\frac{3}{4}$ wide, will line a cloak, containing 9.75 yards of broadcloth, that is 1.5 yards wide? *Ans.* 19.5*yd.*

11. A bankrupt paid 43 cents for every dollar of his debts. How much would he pay on a debt of $569.31?

12. If 49 men can build a wall in 25.2 days, in what time will 63 men build it? *Ans.* 19.6 *days.*

13. If the price of 16yd. 3qr. of sheeting is $2.68, what is the price of 4.9yd.?

14. A., B., and C. invested $16329.00 in merchandise. They sold their goods so as to gain $4101.375. How much

of the gain ought B. to receive, he having contributed \$7075.15? *Ans.* \$1777.07 +.

15. If 7 cords of wood cost \$38.50, what will 9.5 cords cost?

16. If 11 lb. of butter cost \$2.75, what will 29lb. cost?

17. In what time will \$800 yield \$187 interest, at 6 per cent. per annum? *Ans.* 3*yr.* 10*mo.* 22*dy.*

18. If 4 men can reap 15 acres of grain in 6 days, how many men must be employed to reap 85 acres in 8 days? *Ans.* 17 *men.*

19. If 6 men in 8 days can build a wall 24 yards in length, how many additional men must be employed to continue the wall 48 yards further in 12 days? *Ans.* 2 *men.*

20. If a quantity of provisions will serve 1400 men 20 weeks, at the rate of 14oz. per day to each man, how many men will the same provisions maintain for 32 weeks, if each man is allowed 7 ounces per day? *Ans.* 1750 *men.*

21. A difference of 15° in longitude, makes an hour's difference in time. What is the difference of time between Boston, which is 71° 4′ 20″ W. long., and Washington, which is 77° 1′ 30″ W. long.?* *Ans.* 23*min.* 48$\frac{2}{3}$*sec.*

22. If 11 yards of flannel are worth \$2.31, what will be the price of 19 yards?

23. If a cistern discharges 75.09 gallons of water in 1.5 hours, how much will it discharge in 7.93 hours? *Ans.* 396.9758*gal.*

24. A man paid \$7.98 for the interest of a certain sum

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* United States Almanac.

of money, for 2.1 years. How much would he pay for the use of the same sum, 5.3 years?

25. How far would a ship sail in 11.6 hours, at the rate of 78.1 miles in 14.2 hours?

26. What must I pay my gardener for 39 days' labor, his wages being $10.50 per week?

27. If 12 horses eat 11.375 bushels of oats in a week, how many bushels will 74 horses eat in the same time?
Ans. $70\frac{7}{48}$*bu.*

28. How many hours will it take a carrier-dove to fly 190.95 miles, at the rate of 44.25 miles in 2.95 hours?

29. The difference in time between London and New York, is 4h. 56m. 4.5sec. What is the difference of longitude?* [See Ex. 21.] *Ans.* 74° 1′ 7.5″.

30. How many men will build a wall in 13.174 days, that 14 men can build in 9.41 days?

31. How many dollars are equivalent to £13 7s. 11d.; $1.00 being equivalent to 4s. 6d.? *Ans.* $59.537 +

32. How many men can do a piece of work in 28.75 days, that 39 men can do in 57.5 days?

33. The debts of a bankrupt are $19946.25, to meet which, he has property valued at $7602.375. How much can he pay on a debt of $693.50? *Ans.* $264.32 +.

34. A. & B. traded in company. A.'s money, which was $9621.75, remained in the firm 11.8mo. How long was B.'s money, which was $7569.11, in the firm, the gain being divided equally?

35. In 1830, there were 308 day-schools in New Holland, the maintenance of which cost £13292.† What would it cost to support 900 schools, at the same rate?

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* American Almanac.

† Murray.

36. If the wages of 24 men, for 3.5 days, are $96, what will be the wages of 19 men for 7 days?

37. If 8 men can build a wall 100ft. long, 2ft. wide, and 6ft. high, in a given time, how much wall that is 3ft. wide and 9ft. high, can 27 men build in the same time?

Ans. 150*ft.*

38. If 7 men can reap 115.5 acres of wheat in 8.25 days, how many men can reap 276.3 acres in 7.675 days?

39. If the interest of $231.00, for 1yr. 5mo., is $19.635, what will be the interest of $500.00 for 3yr. 11mo., at the same rate?

40. If 16 horses eat 96 bushels of oats in 2mo., how many horses will eat 153 bushels in 1 month?

Ans. 51 *horses.*

41. If I pay 17 men $204.00 for 8 days' work, how many men can I hire 19.5 days, for $380.25?

42. If 31.9 acres will pasture 29 oxen 11 weeks, how many acres will pasture 15 oxen 21.4 weeks?

43. My horse can travel 165 miles in 3 days, when the days are 10 hours long. In how many days of 8.75 hours will he travel 192.5 miles? *Ans.* 4 *days.*

44. If I pay $15.00 for the cartage of 9 tons of coal 35 miles, how much must I pay for carting 13 tons 10.5 miles?

45. If 18 men will work 7 days for $157.50, how many days will 13 men work for $97.50?

46. If 12 sacks of sugar, at 7 cents a pound, cost $83.16, what must I give for 16 sacks, at 8 cents a pound?

Ans. $126.72.

47. How many men will be required to perform a piece of work in 7 days, working 9 hours a day, that 21 men can perform in 12 days, by working 8 hours a day?

Ans. 32 *men.*

48. A vessel has provisions sufficient to furnish 30 men for 6 months, allowing each man 18 ounces a day. How long would the same provisions furnish 45 men, allowing 14 ounces per day? *Ans.* $5\frac{1}{7}$*mo.*

49. The pyramid of Cheops is 693ft. square, and 599ft. high.* What is the length of a staff which casts a shadow of 11ft. at the moment when the shadow of the pyramid begins to be visible? *Ans.* $19\frac{1}{63}$*ft.*

50. State 274 × 16 in the form of a proportion, by making 1 the first term.

51. State 6517 ÷ 19 in the form of a proportion, by making 1 either the second or third term.

52. What is the value, in dollars and cents, of the following bill of exchange,—exchange at 5f. 32c. per dollar?

Exchange 10000f. **Second.** Phil., 13th July, 1844.

Sixty days after sight of this, our second of Exchange, (first and third of same tenor and date unpaid,) pay to the order of J. Villiers & Cie., ten thousand francs, for value received, as per advice, for the account of

To Messrs. Noros & Chopin, } THOMPSON & CLARK.
Paris. } *Ans.* $1879.70.

53. In 9450 milrees 320 rees, how many dollars and cents?

54. Reduce 487 roubles 63 copecks, to Federal Money; exchange at 75 cents per rouble.

55. If 29 men can do a piece of work in 11 days, by working 10 hours a day, how long will it take 18 men, working 8 hours a day, to do the same work?

Ans. $22\frac{11}{72}$ *days.*

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* Murray.

56. If 93 barrels of flour cost $418.50, what will 137 barrels cost?

57. When exchange on London is at par, and £9 = $40, what is the value of £27 13s. 11d.? *Ans.* $123.09.

58. How many men will reap 96 acres in 12 days, if 13 men reap 78 acres in 6 days?

59. If I pay $14 for the freight of 3 tons, 44 miles, what must I pay for the freight of 11 tons, 17 miles?

60. What is the value of 19 marcs 12 schillings Hamburg,—exchange at 35 cents per marc? *Ans.* $6.91.

61. The tower of Strasburg cathedral is 470ft. high.* What is its distance from an observer, when a foot rule, placed 2ft. 6in. from the eye, intercepts the rays from the top and base? *Ans.* 1175*ft.*

62. How many men, in 6 months, will build a wall that 36 men will build in 8 months?

63. How many bushels of meal will serve 36 persons 12 months, if 10 persons consume 8 bushels in 2 months? *Ans.* 172.8*bu.*

64. If 18 men build a cistern 20 feet long, 12 feet wide, and 10 feet high, in 3 weeks, by working 5 days in a week, and 9 hours a day, how many men, working 6 days in a week, and 12 hours a day, will build a cistern 32 feet long, 16 feet wide, and 15 feet high, in 12 weeks? *Ans.* 9 *men.*

65. If $12.00 gain $3.00 in 5 months, how much ought $25.00 to gain in 10 months?

66. The difference in time between London and New Orleans, is 6h. 27.3sec.† What is the difference of longitude?

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* Murray.

† American Almanac.

67. If the freight of 900lb., for 56 miles, is $2.50, how far may 2 tons be carried for $40.00?

68. If $1200 will support 24 persons 8 months, how long will $900 support 16 persons?

69. If $59\frac{1}{5}$cwt. of sugar cost $375, what will $6\frac{2}{5}$cwt. cost?

70. The American Fall at Niagara is 165ft. high.* If an observer stands a mile below the Fall, at what distance from his eye must he place a two-foot rule, so as to intercept the rays from the top and foot of the Fall?

Ans. 64*ft.*

71. If the rent of 22A. 2R. of land is £4 10s., what will be the rent of $24\frac{1}{2}$A.?

72. The shadow of a stick that is 5 feet 6 inches high, measures 3ft. 4in. What is the height of a tree whose shadow measures 70 feet at the same time?

73. If the expenses of a family of 8 persons are $40 in 10 weeks, how many persons can be supported $12\frac{1}{2}$ weeks for $100?

74. What is the difference in time between Nantucket and St. Louis, the difference of longitude being $19° 29' 48''$?†

75. How much wheat, at $1.20 a bushel, must be given in exchange for 90 barrels of flour, at $4.80 per barrel?

76. If 10 compositors, in 4 days of 10 hours, set $66\frac{2}{3}$ pages of types, each page containing 45 lines of 50 letters, how many compositors will set $94\frac{2}{7}$ pages, each page containing 35 lines of 40 letters, in $5\frac{1}{2}$ days of 8 hours?

Ans. 8 *compositors.*

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* Murray.

† American Almanac.

77. If $29\frac{1}{3}$ bushels of wheat yield 1760 bushels in 5 years, how much will $15\frac{1}{2}$ bushels yield in 6 years?

78. A crew of 150 men were supplied with provisions for 9 months, allowing each man 2 pounds per day. When they have been out half the expected time, they find it will require 6 months to finish their voyage. How much may be allowed daily to each man, 25 of the crew having been lost? *Ans.* $1\frac{4}{5}$*lb.*

79. The mosque of Omar, which stands on the site of the temple of Solomon, encloses a court 1489ft. long and 995ft. broad.* If the breadth of this court be represented by 5, what number will represent the length?

Ans. $7\frac{96}{199}$.

80. Cleopatra's needle is built of a single stone, 7ft. square at the base, and 60ft. high.† What should be the height of a model which is $1\frac{1}{2}$ inches square at the base?

Ans. $12\frac{6}{7}$ *inches.*

81. If $42\frac{1}{2}$ bushels of corn, that weighs $51\frac{1}{4}$ pounds a bushel, can be bought with 23 bushels of wheat, that weighs $56\frac{3}{4}$ pounds a bushel, how much corn, weighing 60 pounds a bushel, would be equivalent to 100 bushels of wheat that weighs 54 pounds a bushel?

Ans. 150*bu.* 6*qt.*

82. If a man travels 240 miles in 8 days, when the days are 12 hours long, how many miles will he travel in 24 days, when the days are 16 hours long?

83. If the freight of 2T. 6cwt., for 28 miles, is $14.50, what will be the freight of 9T. 4cwt. for 96 miles?

Ans. $198.86.

84. The Alexandrian library is said to have contained 700000 volumes. What would have been the value of the

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* Murray.

† Pinnock.

whole, provided the average value was $137.50 for each 112 volumes? *Ans.* $859375.

85. If 4 men, in 3 days of 8 hours, build 40 rods of wall, how many rods will 18 men build in 5 days of 9 hours?

86. How many men, in 24 days of 16 hours, will do three times as much work as 18 men can perform in 32 days of 12 hours? *Ans.* 54 *men.*

87. St. Peter's church, at Rome, is 300ft. high.* What is the length of its shadow, when a staff 7ft. high casts a shadow of 15ft. 9in.?

88. If 11 men reap 16 acres of grain in $3\frac{1}{2}$ days, how many acres can 7 men reap in 15 days?

Ans. $43\frac{7}{11}$ *acres.*

89. If 14 men, in $5\frac{1}{2}$ days, by working 8 hours a day, reap $38\frac{1}{2}$ acres of grain, how many men will reap $37\frac{1}{2}$ acres in $6\frac{2}{3}$ days, by working 9 hours a day? *Ans.* 10 *men.*

90. The length of a degree of longitude on the 40th parallel of latitude, is 46 miles.† How many miles must one travel on that parallel, to make a difference of 57 minutes in time, every degree making a difference of 4 minutes? *Ans.* $655\frac{1}{2}$ *miles.*

91. If 36 men, in $127\frac{1}{2}$ days of $13\frac{1}{2}$ hours, dig a trench $33\frac{3}{4}$yd. long, $10\frac{1}{2}$ft. deep, and $15\frac{3}{5}$ft. wide, how many men, in $7\frac{2}{7}$ days of $12\frac{8}{11}$ hours, will dig a similar trench $82\frac{8}{11}$yd. long, $7\frac{2}{5}$ft. deep, and 10ft. wide? *Ans.* 740 *men.*

92. According to Herodotus, the pyramid of Cheops required the constant labor of 100000 men for 20 years. How many men would have been required to build it in 125 years?

93. The population of Rome, under Aurelian, has been

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* Pinnock.

† Willard.

estimated at 3000000.* How many tons of provisions would be consumed daily by such a population, provided the average weekly consumption was 14lb. to each individual?

94. The velocity of wind in a hurricane is sometimes 100 miles per hour, or 146$\frac{2}{3}$ft. per second.† What would be the velocity per second in a severe storm, when the velocity was 60 miles per hour? *Ans.* 88*ft.*

95. The Parthenon, or Temple of Minerva, at Athens, was 217ft. long, 98ft. broad, and 65ft. high.* If an exact model were made, 3ft. in length, what would be the breadth and height?

96. The Pharos, or watch-tower of Alexandria, which was called one of the seven wonders of the world, was 400ft. high.* At what distance from its base would it have appeared of the same height as a 6ft. pole placed 27ft. from the eye? *Ans.* 1800*ft.*

97. The length of St. Peter's church is 840ft.‡ What should be the length of an exact model that is 9 inches high? What should be the breadth of such a model, the breadth of the church being 725ft.?‡ [See Ex. 87.]

98. What should be the base circumference of the model referred to in the preceding example, the circumference of the church being 2465ft.?‡

99. By an account taken under Demetrius Phalereus, soon after the time of Alexander, the inhabitants of Athens amounted to 440000, of whom much the greater part were slaves.§ If there were 7 slaves to every 4 free citizens, what was the entire number of slaves? *Ans.* 280000.

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* Encyclopædia Americana.

† London Philosophical Transactions, vol. 51.

‡ Pinnock.

§ Willard.

100. The Manchester and Liverpool Railway is 31 miles long, and cost £820000.* What would it cost to build a road 100 miles long, at the same rate?

Ans. £2645161 5*s.* 10*d.*

101. The annual tribute levied on the satrapy of India, by the Persian emperors, was 4680 talents of silver.* To what was this sum equivalent, estimating the value of 13 silver talents at $26280?†

102. The annual tribute paid by the satrapy of Egypt, including Cyrene and Barca, was 1400 talents of silver, one-half of which was paid in grain. Required its value in Federal Money.

103. The annual tribute of the satrapy of Babylonia and Assyria, including Syria, was 1000 talents of silver. Find its value in our currency.

104. The entire annual tribute levied on the satrapies of the Persian empire, was 13130 silver talents. Reduce the amount to sterling money, estimating the pound sterling at $4.83. *Ans.* £5495403 14*s.* 6*d.*

105. About 2000 years ago, Eratosthenes estimated the circumference of the earth to be 252000 stadia.* According to this estimate, how many stadia were equivalent to a degree?

106. The annual value of the woollen manufacture of Great Britain has been estimated at £20000000.* To how many yards would this be equivalent, at the average rate of £5 for every 11 yards?

107. The annual value of the cotton manufacture of Great Britain has been estimated at £34000000.* To how

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* Murray.

† The value of the ancient coins can only be estimated approximately. The estimate here given, is very nearly that of McCulloch.

many yards would this be equivalent, at the average rate of £2 for every 97 yards?

108. The spire of Salisbury cathedral is 410ft. high.* At what distance from the observer would a tree 50ft. high appear of the same height as the cathedral at a distance of 100 yards? *Ans.* $12\frac{8}{41}$ *yards.*

109. The number of deaths in England, in 1780, was in the proportion of 25 to every 1000 of the entire population.* At this rate, what would be the annual mortality in a city of 27840 inhabitants? *Ans.* 696.

110. The number of deaths in England, in 1830, was in the proportion of 5 to every 270 of the entire population.* At this rate, how much less would be the annual mortality in a city of 58320 inhabitants, than at the rate of 1780?

111. The population of England, during the reign of Elizabeth, was estimated at $4\frac{1}{2}$ millions.* What was the annual number of deaths, if the mortality was at the rate of 13 for every 480 inhabitants?

112. The quantity of iron annually manufactured in Wales has been estimated at 270000 tons, requiring a consumption of 11 tons of coal for every 2 tons of iron.* How many tons of coal are required for the whole?

113. Scotland contains 18944000 acres of land, of which only 5043000 are regularly cultivated, and not more than 1800000 are under grain.* At this rate, what is the average number of acres under cultivation, and the number under grain, in each square mile?

Ans. $170\frac{55}{148}$ *acres under cultivation.*
$60\frac{30}{37}$ *acres under grain.*

114. The coal field of Connaught contains 114000 Irish

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* Murray.

acres.* To how many standard acres is this equivalent, 121 Irish acres being equal to 144 standard acres?

Ans. 135669$\frac{51}{121}$ *standard acres.*

115. In the Netherlands, in the year 1827, the proportion of children at school to the entire population, was as 2 to 19.* At the same rate, how many school-children would there be in the United States, according to the census of 1850, when the population was 23,387,125?

116. The cathedral of Antwerp is 500ft. long, 230ft. wide, and 360ft. high, with a spire 466ft. high.* What should be the dimensions of a model, the length of which is 4ft.?

117. Floats of timber come down the Rhine, so large that one of them has been valued at 350000 florins.* Reduce this amount to Federal Money, estimating at the rate of $4.80 for every 12 florins.

118. The average proportion of male to female births in France, is as 16 to 15.* At this rate, out of 57970 children, how many would be boys, and how many girls?

Ans. 29920 *boys;* 28050 *girls.*

119. The fisheries of Calais vary in value, from 1000000 to 2000000 francs.* Express these amounts in Federal money, estimating the 5-franc piece at 93 cents, which is the Custom House valuation.

120. In Westminster Hall, which is 276ft. long by 76ft. broad, 10000 persons have dined at one time.* How many could dine with equal accommodations in a hall 115ft. long and 57ft. broad?

Ans. 3125.

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* Murray.

XIV. FELLOWSHIP, OR PROPORTIONAL DIVISION.

112. SINGLE FELLOWSHIP.

BY the Rule of Followship, we divide quantities into parts proportioned to certain given numbers. This division is required in determining the gain or loss of partners in trade or joint adventures, the worth of debts in bankruptcy, the adjustment of legacies when the property of the testator is not sufficient to pay the whole amount of the legacies, &c.

In Single Fellowship, the division is made in proportion to the quantities as they are given, no multiplication being required among the given numbers.

113. EXAMPLE FOR THE BOARD.

Three partners in trade gained $262.50. How should the gain be divided, A.'s stock being $1200, B.'s $600, and C.'s $300.

We may consider each dollar as representing a share of stock. Then A. holds 1200 shares, B. 600 shares, and C. 300 shares, making a total of 2100 shares. A. owning $\frac{1200}{2100}$, or $\frac{4}{7}$ of the whole stock, should have $\frac{4}{7}$ of the gain; B., owning $\frac{600}{2100}$, or $\frac{2}{7}$ of the stock, should have $\frac{2}{7}$ of the gain; C., owning $\frac{300}{2100}$, or $\frac{1}{7}$ of the stock, should have $\frac{1}{7}$ of the gain. $\frac{4}{7}$ of \$262.50 = \$150, *A.'s share.* $\frac{2}{7}$ of \$262.50 = \$75, *B.'s share.* $\frac{1}{7}$ of \$262.50 = \$37.50, *C.'s share.*

A.,	1200
B.,	600
C.,	300
Total,	2100

The rule for this operation may be stated in several different ways, each of which requires the same operations on the given numbers.

I. *Solve the question by proportion, according to the following statement: The sum of the proportional numbers* is to *each proportional number* as *the whole quantity to be divided* is to *each proportional part.* Or,

II. *Multiply the whole quantity to be divided by each proportional number, and divide each product by the sum of the proportional numbers.* Or,

III. *Divide the whole quantity by the sum of the proportional numbers, and multiply the quotient, (reduced to its lowest terms,) by each proportional number.*

PROOF. *Add all the shares, and if they are correctly determined, their sum will be equal to the quantity which was to be divided.*

[The third of the above methods is probably the most convenient in general practice.]

114. EXAMPLES IN SINGLE FELLOWSHIP.

1. A., B., and C. entered into partnership. A. contributed \$500, B. \$300, and C. \$600. At the end of a year they had gained \$350. What was the share of each?
Ans. A.'s, \$125; *B.'s,* \$75; *C.'s,* \$150.

2. The debts of a bankrupt amount to \$1275, and his whole property is worth but \$950. What proportion of the whole can he pay, and what is the value of a debt of \$361.25? *Ans.* $\frac{38}{51}$; \$269.17.

[The sum of the proportional quantities is the amount of the debts. The value of only one of the parts is required.]

3. A vessel during a storm sustained a damage of \$1950. How much of the loss should be sustained by A., whose share of the cargo was \$1000, the whole value of the vessel and cargo being \$21500?

[The sum of the proportional numbers is the whole value of the vessel and cargo.]

4. William Johnson, Samuel Buffum, and Moses Clark entered into partnership. Johnson contributed \$4750; Buffum, \$5250; and Clark, \$6000. At the end of three years, they had lost \$2500. What was the loss of each?

5. A., B., and C. traded in company. A. put in \$3000, B. \$2000, and C. the remainder. They gained \$1500, of which C. took \$500. What did C. contribute, and what was A.'s share of the gain?

[\$1000 are left to be divided between A. and B. As C. receives $\frac{1}{2}$ as much as A. and B. together, his contribution must have been $\frac{1}{2}$ as great as the sum of theirs.]

6. The property of a bankrupt is worth but \$5163.90, and he owes A. \$1765.75, B. \$840.00, C. \$2329.50, and D. \$1380.10. How much will each creditor receive?

Ans. *A.*, \$1443.81; *B.*, \$686.85; *C.*, \$1904.77; *D.*, \$1128.47.

7. A. contributed \$380 to an adventure, B. \$420, C. \$400, and D. \$800. What was each man's share of the gain, which was \$900?

8. Divide 1200 into shares proportioned to the numbers 1, 2, 3, and 4. *Ans.* 120; 240; 360; 480.

9. Divide 768 into shares proportioned to the numbers 2, 5, and 13.

10. Divide 4000 into shares proportioned to the numbers $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{2}{3}$. *Ans.* $1043\frac{11}{23}$; $1565\frac{5}{23}$; $1391\frac{7}{23}$.

11. The distance from the sun to the orbit of Mercury, is 37 million miles; from the orbit of Mercury to the orbit of Venus, 31 million miles; from the orbit of Venus to the orbit of the earth, 27 million miles; from the orbit of the earth to the orbit of Mars, 49 million miles; from the orbit of Mars to the orbit of Jupiter, 346 million miles; from the orbit of Jupiter to the orbit of Saturn, 410 million miles; from the orbit of Saturn to the orbit of Uranus, 900 million miles; from the orbit of Uranus to the orbit of Neptune, 1080* million miles. If an orrery were made, in which Neptune was placed 4 feet from the sun, at what

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* S. C. Walker.

distance from each other should the several planets be placed?

12. A grocer finds that by mixing 3lb. of sugar at 12 cents a pound, 1 lb. at 10 cents, 1 lb. at 8 cents, and 3lb. at 6 cents, he can form a mixture worth 9 cents a pound. How much of each kind should he take to make 260lb. of the mixture?

Ans. 97½*lb.* at 12*ct.*; 32½*lb.* at 10*ct.*; 32½*lb.* at 8*ct.*; 97½*lb.* at 6*ct.*

13. According to the census of 1840, the population of Maine was 501793; New Hampshire, 284574; Vermont, 291948; Massachusetts, 737699; Rhode Island, 108830; Connecticut, 309978.* If $1000000 were to be divided among the New England States, in proportion to their population at that time, what would be the share of each?

14. The value of a vessel and cargo is $14000; of which A. insures $3250; B., $1875; C., $900; D., $700; and E., $1225. A loss being sustained to the amount of $6250, how much must each of the underwriters† pay, and how much must be borne by the proprietors?

Ans. *A.*, $1450.89; *B.*, $837.05; *C.*, $401.79; *D.*, $312.50; *E.*, 546.88; *Proprietors*, $2700.89.

15. The amount of the legacies bequeathed in a will was $35000, but the whole amount of the testator's property was found to be only $31500. What was the value of the wife's legacy, which was $10000, and of the eldest son's, which was $15000?

16. It is required to divide $36000 among 4 persons, in such manner that the second may have twice as much as the first, the third as much as the first and second, and the fourth three times as much as the third.

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* American Almanac.

† Insurers are sometimes called *underwriters.*

17. A legacy of $40000 was left to four heirs, in the proportion of $\frac{1}{6}$ to the first, $\frac{2}{5}$ to the second, $\frac{4}{9}$ to the third, and $\frac{1}{3}$ to the fourth. What was the share of each?

Ans. 1*st,* $4958.68; 2*d,* $11900.82; 3*d,* $13223.14; 4*th,* $9917.36.

18. A levy of 1240 horses is to be distributed among three regiments, in the proportion of 1, $\frac{8}{11}$, and $\frac{7}{9}$ respectively. How many will each regiment receive?

Ans. 495; 360; 385.

19. A bankrupt owes $150000, to pay which he has the following property: real estate worth $17500; merchandise worth $3750; personal property worth $20000, and sums due from various individuals amounting to $100000. If all his dues are collected, how much must be paid on every dollar of his debts?

20. A., B., and C. purchase a house for $10000, A. contributing $4000, B. $3600, and C. $2400. The house rents for $800, and the taxes and repairs amount to $50 a year. What income does each of the owners derive from the estate, and what per cent. of his investment?

Partial Ans. $7\frac{1}{2}$ *per cent.*

21. Four men were joint owners of a farm, their shares being in the proportion of 6, 2, 5, and 3, respectively. It is required to divide the annual rent, which is $1500, equitably among them.

115. COMPOUND FELLOWSHIP.

In Compound Fellowship, the division of quantities is made in a compound ratio, which ratio is determined by multiplying certain given numbers together.

For example, in partnership, when the stocks of the different partners are engaged in trade for unequal times, the proportional numbers for the division of the gain or loss, may be found by multiplying each man's stock by the time of its employment.

116. EXAMPLE FOR THE BOARD.

Three persons hired a meadow, for which they paid $45. A. put in 90 sheep for 12 days; B. put in 80 sheep for 25 days; C. put in 60 sheep for 32 days. What part of the cost must be paid by each person?

$$\begin{array}{r} 90 \times 12 = 1080 \\ 80 \times 25 = 2000 \\ 60 \times 32 = 1920 \\ \hline 5000 \end{array}$$

The pasturage of 90 sheep for 12 days, would be equivalent to the pasturage of 12 × 90 sheep for 1 day. The pasturage of 80 sheep for 25 days, would be equivalent to the pasturage of 80 × 25 sheep for 1 day. The pasturage of 60 sheep for 32 days, would be equivalent to the pasturage of 60 × 32 sheep for 1 day. The entire expense should, therefore, be shared in the same proportion as if A. had pastured 1080 sheep, B. 2000 sheep, and C. 1920 sheep, for equal times. Hence we deduce the General Rule for Compound Fellowship.

Form the continued product of each set of proportional numbers, as in Compound Proportion. Work with these several products, as in Single Fellowship.

117. EXAMPLES IN COMPOUND FELLOWSHIP.

1. A. contributed $300 for 5 months, B. $700 for 2 months, C. $1000 for 1 month. They gained $78. How should it be divided?

Ans. A., $30; *B.*, $28; *C.*, $20.

2. Three brothers send children to a boarding-school. A. sends 2 children for 6 months, B. sends 3 for 3 months, C. sends 5 for 9 months. Their entire bill is $550. How much must each pay?

Ans. A., $100; *B.*, $75; *C.*, $375.

3. In a certain firm A. contributes $5000 for 4 months,

B. $6000 for 3 months, C. $2000 for 6 months, D. $4000 for 5 months, and E. $5000 for 2 months. How should the gain, which was $1680, be divided among the partners?

Ans. A., $420; *B.*, $378; *C.*, $252; *D.*, $420; *E.*, $210.

4. A. and B. form a limited partnership for 2 years and 6 months. A. at first contributed $2000, and in 13 months $1000 more. B. contributed at first $2500, and at the end of the first year $600 more, but at the end of 19 months he withdrew $2000. At the end of 14 months, C. was admitted to the firm, and contributed $3700. How should the gain, which was $3000, be divided?

[A.'s share is represented by (2000 × 13) + (3000 × 17). B.'s share, by (2500 × 12) + (3100 × 7) + (1100 × 11). C.'s share, by 3700 × 16.]

5. Three companies of men worked on a railroad. In the first company there were 25 men, who worked 8 days and 10 hours a day; in the second, there were 27 men, who worked 11 days and 12 hours a day; in the third, there were 19 men, who worked 18 days and 11 hours a day. What were the wages of each company, their entire wages being $932.60?

6. A. and B. trade in company. A. contributes at first $800, and B. $1000. At the end of 3 months, A. advances $1200 more. What sum must B. add to the firm at the end of 6 months, to entitle him to one-half of the year's profits?

[A.'s share is represented by (800 × 3) + (2000 × 9). B. must add, at the end of 6 months, such a sum that the products which represent his share, will amount to the same as the products which represent A.'s share.]

7. Five men hire a pasture for $294. A. put in 10 cows for 3 months; B. put in 5 cows for 2 months; C. put

in 8 cows for $1\frac{1}{2}$ months; D. put in 16 cows for $2\frac{1}{4}$ months; and E. put in 20 cows for 4 months. How much ought each to pay?

Ans. *A.*, $52.50; *B.*, $17.50; *C.*, $21; *D.*, $63; *E.*, $140.

8. A., B., and C. form a joint stock of $1098, by which they gain $234. A.'s money is in trade 4 months, B.'s 5 months, and C.'s 13 months, and their shares of the gain are as the numbers 2, 3, and 4, respectively. What were each man's stock, and gain?

[The shares of the gain are determined by Single Fellowship. The proportional numbers for determining the shares of stock, may be found by dividing 2, 3, and 4, by 4, 5, and 13, respectively.]

9. A. and B. start from St. Louis and Washington, which are 850 miles apart, and travel at equal rates until they meet. A. travels 4 days for $7\frac{1}{2}$ hours a day, and B. travels 5 days for 9 hours per day. What distance does each man go before they meet, and at what rate per hour do they travel? *Partial Ans.* *A.* 340*m.*; *B.* 510*m.*

10. A. commenced business with a capital of $2500. Five months afterwards he received B. as a partner, who added $4000 to the stock. Six months after B. was admitted, C. joined the firm with a capital of $6000. Two years from the commencement, they had gained $8000. What was each man's share of the gain?

11. The cost of three walls was $495. One of the walls was 40 rods long, 2ft. thick, and 6ft. high; one was 28 rods long, $1\frac{1}{2}$ft. thick, and 5ft. high; and one was 100 rods long, 1ft. thick, and 3ft. high. What was the price of each, they all being built of the same materials and at the same rate?

Ans. $240; $105; $150.

12. Three lots of freight were carried on a railroad. The

first weighed 17cwt., and was carried 20 miles; the second weighed one ton, and was carried 17 miles; the third weighed 11cwt., and was carried 45 miles. They were all charged at the same rate, and the freight of the whole was \$5.87½. What was the freight of each lot?

Ans. \$1.70; \$1.70; \$2.47½.

13. Two books were stereotyped. One contained 240 pages, each page containing 40 lines, and 42 *m*'s* in a line; the other contained 384 pages, each page containing 36 lines, and 40 *m*'s in a line. What was the cost of each, the cost of the two being \$637.44?

Ans. \$268.80; \$368.64.

14. The crews of two vessels number respectively 150 and 125 men. The first are supplied with provisions for 9 months, allowing each man 2lb. per day; the second are supplied with provisions of the same kind for 8 months, allowing each man 2½lb. per day. The whole supply was made by contract for \$7800. How much should be paid by the owners of each vessel? *Ans.* \$4050; \$3750.

15. Three men hired a pasture for \$40. A. put in 4 horses for 3 months and 6 for 2 months; B. put in 5 for 4 months and 10 for 1 month, and C. put in 7 for 4 months and 3 for 2 months. How much ought each to pay?

16. A capitalist made four different loans, at the same rate of interest; one of \$540 for 27 days; one of \$360 for 42 days; one of \$288 for 75 days, and one of \$600 for 19 days. What was the interest on each loan, the interest on the whole being \$12.19?

Ans. \$2.83; \$2.94; \$4.20; \$2.22.

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* Printer's work is measured by the thousand ems.

XV. EQUATION OF PAYMENTS, AND AVERAGE.

118. Simple Equation.

When several debts are due at different times, it is often desirable to find some time at which they may all be paid, without loss to either the debtor or the creditor. This time is called the Average, or Equated Time, and the process by which it is found is the Equation of Payments.

The ordinary rule for equation of payments, is founded on the supposition that the interest of the money which is not paid until after it is due, is equal to the discount of that which is paid before it is due. This is not strictly correct, but it corresponds to the usual method of computing discount among merchants.

The labor of equating may be abbreviated by disregarding the cents if they are less than 50, and counting them as an additional dollar if they are more than 50. When the sums are all large, the units of dollars may be disregarded in a similar manner.

To find the equated time for the payment of several debts:

Multiply each charge by the time which has elapsed from the date of the first bill, and divide the sum of the products by the sum of the bills.

119. EXAMPLE FOR THE BOARD.

A merchant owes $4.00, payable in 3 months; $6.00 payable in 6 months, and $2.00 payable in a year. At what time may he pay the whole, so that neither party shall lose any interest?

The interest of $4.00 for 3mo. = The int. of $1.00 for 12mo.
The interest of $6.00 for 6mo. = The int. of $1.00 for 36mo.
The interest of $2.00 for 12mo. = The int. of $1.00 for 24mo.

The interest of $12.00 for —mo. = The int. of $1.00 for 72mo.

He may keep $12, $\frac{1}{12}$ as long as $1.00. Therefore, dividing 72mo. by 12, we obtain 6mo. for the equated time.

120. EXAMPLES FOR THE PUPIL.

1. I have three notes given by James Appleton; one of $75, payable in 30 days; one of $50, payable in 60 days; and one of $500, payable in 90 days. What is the equated time for the payment of the whole?

2. What is the equated time for the payment of $1000, $\frac{1}{4}$ of the sum being now due, $\frac{1}{4}$ in 3 months, and $\frac{1}{2}$ in 9 months? [The $250 now due has no time to run, and the product $250 \times 0 = 0$.]

3. What is the equated time for the payment of three debts, one of $100, due in 8mo.; one of $25, due in $1\frac{1}{4}$yr.; and one of $90, due in 2yr.?

4. I owe $40, payable now; $40 in 30 days; $20 in 60 days, and $100 in 120 days. What is the equated time for the payment of the whole?

5. The following charges are made against Walton & Morris, each payable in 8 months, viz: Jan. 25, $1643.50; Feb. 1, $900; Feb. 16, $1131.25; Feb. 28, $241.75; March 13, $971; and April 3, $763.37. What should be the date of a single note at 8 months, for the whole amount?
Ans. Feb. 18.

6. What should be the date of a note at 60 days, (or 63 days, including the days of grace,) for $150, due April 7; $200, due April 13; $75, due April 17; $325, due April 29; $180, due April 30; $400, due May 4, and $439, due May 11?
Ans. Feb. 25.

121. EXAMPLES IN AVERAGE.

The average price of a number of ingredients of different values, is found in nearly the same way as the average time for the settlement of a number of debts by a single payment.

1. Required the average length of 5 pieces of sheeting, measuring as follows: No. 1, $32\frac{1}{8}$yd.; No. 2, $31\frac{1}{2}$yd.; No. 3, $33\frac{1}{2}$yd.; No. 4, $30\frac{3}{4}$yd.; No. 5, 34yd. *Ans.* $32\frac{3}{8}$yd.

2. A grocer mixes 10 pounds of tea, at 50 cents a pound, 12 pounds at 75 cents, and 16 pounds at $62\frac{1}{2}$ cents. What is the value of a pound of the mixture?

3. If the thermometer stands from 6 to 9 at 68°, from 9 to 11 at 70°, from 11 to 1 at 73°, from 1 to 4 at 77°, and from 4 to 6 at 76°, what is the average temperature of the 12 hours? *Ans.* $72\frac{3}{4}$°.

4. A grocer mixed 18 gallons of molasses at 25 cents a gallon, with 20 gallons at 30 cents, 24 gallons at 40 cents, and 5 gallons of water. At what price per gallon must he sell the mixture to gain 25 per cent.?

5. If I mix 8lb. of gold 22 carats fine, 6lb. 20 carats fine, 8lb. 24 carats fine, 10lb. 23 carats fine, and 2lb. of alloy, what will be the fineness of the mixture? [A carat is $\frac{1}{24}$. Pure gold is 24 carats fine. In gold that is 22 carats fine, $\frac{22}{24}$ of the whole is pure gold. Alloy is counted of no value.] *Ans.* $21\frac{2}{17}$ *carats.*

6. A jeweller mixed 3lb. of silver, 9oz. fine, 4lb. of 11oz. fine, 2lb. of $10\frac{1}{2}$oz. fine, and 2lb. of 10oz. fine. What was the fineness of the mixture? [11oz. fine, denotes that 11oz. in the lb. are silver, and the rest alloy. Pure silver is 12oz. fine.]

122. COMPOUND EQUATION.

To find the equated time for the settlement of an account, in which there are both debits and credits:—

1st. *Find the equated time for each side of the account.*

2d. *Multiply the least side of the account by the time between the dates found for each side, and divide the product by the balance of the account. The quotient will be the time between the date found for the larger side of the account, and the equated time for the settlement of the balance.*

If the date found for the larger amount is the earliest, count back, but if it is the latest, count forward from that date.

123. EXAMPLE FOR THE BOARD.

Required the time when the balance of the following account became due:

Dr. *Martin Acres, in Account with Simon Gray.* *Cr.*

1849				1849			
June 14	To balance	$875	30	July 10	By Cash	$600	00
Aug. 18	" Mdse.	519	40	Sept. 16	" Mdse.	427	90
Sept. 22	" Bill due	289	37	Nov. 18	" Draft	350	00
Dec. 18	" Do.	318	00	Dec. 20	" Cash	275	00

Drs.	*Crs.*
875.30 × 0 = 0	600.00 × 26 = 15600.00
519.40 × 65 = 33761.	427.90 × 94 = 40222.60
289.37 × 100 = 28937.	350. × 157 = 54950.
318. × 187 = 59466.	275. × 189 = 51975.
2002.07 122164.	1652.90 162747.60
122164 ÷ 2002.07 = 61.	162747.60 ÷ 1652.90 = 98.

Debits due 61 days after June 14. Credits due 98 days after June 14.

1652.90 × 37 ÷ 349.17 = 175. The balance was therefore due 175 days before the date found for the Dr. side, or 114 days before June 14, which was Feb. 20.

The example may be equated as follows, by disregarding the cents, and units of dollars. If the dollars amount to 5 or more, 1 should be added to the number of eagles.

$$\begin{array}{rcrcr} 88 & \times & 0 & = & 0 \\ 52 & \times & 65 & = & 3380 \\ 29 & \times & 100 & = & 2900 \\ 32 & \times & 187 & = & 5984 \\ \hline 201 & & & & 12264 \end{array} \qquad \begin{array}{rcrcr} 60 & \times & 26 & = & 1560 \\ 48 & \times & 94 & = & 4042 \\ 35 & \times & 157 & = & 5495 \\ 28 & \times & 189 & = & 5292 \\ \hline 166 & & & & 16389 \end{array}$$

12264 ÷ 201 = 61.   16389 ÷ 166 = 98.

1652.90 × 87 ÷ 849.17 = 175, as before.

If accounts are not settled at the time they become due, the debtor is either charged, or credited, with interest. If settled before they are due, interest is credited; if settled after they are due, interest is charged.

124. EXAMPLES FOR THE PUPIL.

1. The debits of an account are as follows: June 14, $800; Aug. 18, $500. There is a single credit, July 18, $1000. Find the equated time for the balance.

Ans. June 10.

2. Find the equated time for the settlement of the following account: Debits, Jan. 6, $400; March 10, $800; May 20, $250. Credits, Feb. 20, $700; April 10, $100; July 5, $100. *Ans. Feb.* 22.

3. When was the balance of the following account due? Debits due, Jan. 1, 1850, $800; May 15, $250; July 5, $500. Credits due, Jan. 12, $400; April 3, $500; June 29, $50. *Ans. April* 8.

4. Average the following account. Debits, March 3, $200; April 10, $100; May 1, $150; June 8, $200. Credits, March 25, $300; May 16, $75; June 20, $100.

Ans. April 24.

XVI. ALLIGATION.

125. When the values of a variety of ingredients are given, and it is desired to make a mixture of any fixed value, the quantity of each ingredient to be employed is determined by ALLIGATION.

The name "Alligation," signifies a binding, or tying together; the usual rules for solving questions in alligation, requiring that the quantities should be connected by lines. But as these rules give only a few of the many answers which may be obtained, the following general rule is preferable.

Assume any quantity you please of each ingredient, find the cost of the whole, and also the cost of the same quantity at the mean rate proposed. If the assumed quantities cost TOO MUCH, *take such additional quantities of the lower priced, or such diminished quantities of the higher priced ingredients, as will exactly counterbalance the excess. If* TOO LITTLE, *take such additional quantities of the higher priced, or such diminished quantities of the lower priced ingredients, as will exactly counterbalance the deficiency.*

126. EXAMPLE FOR THE BOARD.

In what proportion should I mix sugars at 5ct., 7ct., 8ct., 10ct., and 12ct., in order that the mixture may be worth 9ct. a pound?

3 lb.	at 5ct.	=	.15
1 lb.	at 7ct.	=	.07
2 lb.	at 8ct.	=	.16
4 lb.	at 10ct.	=	.40
5 lb.	at 12ct.	=	.60
15 lb.	cost		1.38
15 lb.	at 9ct.	=	1.35
Excess			.03

We may commence by taking any quantity we please of each ingredient, and finding the cost of the whole. If, for example, we take 3lb. at 5ct., 1 lb. at 7ct., 2lb. at 8ct., 4lb. at 10ct., and 5lb. at 12ct., (making 15lb. in all,) the whole cost will be $1.38. But 15lb. at 9ct. would cost only $1.35, therefore our estimated quantities give an excess of 3 cents above the required cost. To balance this excess, we must either add some of the sugar that costs less than the average price proposed, or take out some of that which costs more than the average. For every pound that we add at 5 cents, there will be a deficiency of 4 cents from the mean rate; for every pound at 7ct., a deficiency of 2ct.; for every pound at 8ct., a deficiency of

1ct. Then we may either add ¾ of a lb. more at 5ct., or 1½lb. more at 7ct., or 3lb. more at 8ct., or 1 lb. more at 8ct. and 1 lb. at 7ct., or any other quantity that will make a deficiency of 8ct.

If the quantities first assumed had given a *deficiency* instead of an *excess*, we should have been obliged to take some additional quantity of one or more of the ingredients whose value is *greater* than the proposed average, or to take out some of the ingredients whose value is less than the average. The cost of the quantities assumed in the margin, would be only 91ct., but 12lb. at 9ct. would cost $1.08. There is, therefore, a deficiency of 17ct., which may be balanced by taking enough of the sugar that costs more than 9ct., to make an excess of 17ct. Thus, 5lb. additional at 12ct., and 2lb. at 10ct., would answer the required conditions. The pupil may determine other values for himself.

4 lb.	at 5ct.	=	.20
3 lb.	at 7ct.	=	.21
2 lb.	at 8ct.	=	.16
1 lb.	at 10ct.	=	.10
2 lb.	at 12ct.	=	.24
12 lb.	cost		.91
12 lb.	at 9ct.	=	1.08
	Deficiency		.17

The following are therefore some of the answers to the proposed question:—(1.) 8¾lb. at 5ct., 1 lb. at 7ct., 2lb. at 8ct., 4lb. at 10ct., and 5lb. at 12ct. (2.) 3lb. at 5ct., 2½lb. at 7ct., 2lb. at 8ct., 4lb. at 10ct., and 5lb. at 12ct. (3.) 3lb. at 5ct., 1 lb. at 7ct., 5lb. at 8ct., 4lb. at 10ct., and 5lb. at 12ct. (4.) 3lb. at 5ct., 2lb. at 7ct., 3lb. at 8ct., 4lb. at 10ct., 5lb. at 12ct. (5.) 4lb. at 5ct., 3lb. at 7ct., 2lb. at 8ct., 3lb. at 10ct., 7lb. at 12ct.

Let the pupil perform the above examples by *taking out* some of the quantities originally assumed.

127. EXAMPLES FOR THE PUPIL.

1. How much tea at 50ct., 75ct., 90ct., and $1.00 must be taken to form a mixture worth 80ct.?

2. A jeweller has gold of 16, 17, 18, 20, 22, and 24 carats fine. What proportion of each must he take to make a mixture 21 carats fine?

3. How much grain, at 50ct., 75ct., $1.00, and $1.10 per bushel, will make a mixture worth 90ct. a bushel?

4. What quantity of raisins, at 10ct., 18ct., and 20ct per lb., must be mixed together, to fill a cask containing 150lb., and to be worth 19ct. a lb.? [After obtaining the proportions by Alligation, find the exact quantities by Fellowship.]

5. It is required to mix sugar at 7ct., 8ct., 10ct., and 12ct. per lb., in such manner as to form a mixture of 2cwt. 3qr., worth 11ct. per lb.

6. Mix sugar at 8ct., 10ct., 12ct., and 16ct., so as to make 100 pounds worth 11ct. a pound.

7. How much water must be mixed with 40 gallons of syrup, at 50ct. a gallon, to make the whole worth 37½ct. a gallon?

8. How much molasses at 50ct., and water, must be mixed with 15 gallons at 37½ cents, 28 gallons at 25 cents, and 19 gallons at 33 cents, to make a mixture worth 31 cents a gallon?

9. A grocer has an order for 150lb. of tea, at 90 cents per lb., but having none at that price, he would mix some at 75ct., some at 87½ct., and some at $1.00 per pound. How much of each sort must he take?

XVII. INVOLUTION AND EVOLUTION.

128. INVOLUTION.

INVOLUTION is the repeated multiplication of a number by itself.

The product obtained by Involution is called a *power*. The *root* is the number involved, or the *first power*. If the root be multiplied by itself, or employed twice as a factor, the product is the second power. If the root is employed three times as a factor, it is raised to the 3d power; if 5 times, to the 5th power, &c. Thus, 2 is the 1st power of 2, or 2^1. 2×2 or 4, is the 2d power of 2, or 2^2. $2 \times 2 \times 2$ or 8, is the 3d power of 2, or 2^3. $2 \times 2 \times 2 \times 2 \times 2$ or 32, is the 5th power of 2, or 2^5. The power is usually denoted by a small figure over the right of the root, called the *exponent*, or *index*. When there is no exponent, the number is regarded as the 1st power.

The second power is often called the square, because the number of square feet in any square surface is obtained by multiplying the number of feet in one side by itself.

The third power is often called the cube, because the number of cubic feet in any cubical block may be obtained by raising the number of feet in one side to the 3d power.

The 4th power is sometimes called the bi-quadrate, or the square squared; the 5th power, the first sursolid; the 6th power, the square cubed, or the cube squared; the 7th power, the second sursolid; the 8th power, the bi-quadrate squared; the 9th power, the cube cubed; the 10th power, the 1st sursolid squared, &c.

If the exponents of any two powers of the same number be added, we shall obtain the exponent of their product. Thus $6^3 \times 6^5 = 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^8$; $4^2 \times 4^3 = 4 \times 4 \times 4 \times 4 \times 4 = 4^5$.

In any two powers of the same number, if we subtract the smaller exponent from the larger, we shall obtain their quotient. Thus $6^8 \div 6^5 = \frac{6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6}{6 \times 6 \times 6 \times 6 \times 6} = 6 \times 6 \times 6 = 6^3$.

We may represent any power of a number, by multiplying its exponent. Thus, the 7th power of 5 is 5^7; the 3d power of 2^2 is 2^6, because $2^2 \times 2^2 \times 2^2 = 2^6$. These properties form the basis of the system of Logarithms.

129. EXAMPLES FOR THE BOARD.

1. Find the 4th power of 9.7.

$$\begin{array}{r l}
9.7 & = \text{The root, or 1st power.}\\
9.7 & \\
\hline
679 & \\
873 & \\
\hline
94.09 & = 9.7 \times 9.7\text{, the square, or 2d power.}\\
9.7 & \\
\hline
65863 & \\
84681 & \\
\hline
912.673 & = 9.7 \times 9.7 \times 9.7\text{, the cube, or 3d power.}\\
9.7 & \\
\hline
6388711 & \\
8214057 & \\
\hline
8852.9281 & = 9.7 \times 9.7 \times 9.7 \times 9.7\text{, the 4th power.}
\end{array}$$

2. What is the product of $17^5 \times 17^9$?

If 17 is employed 5 times as a factor, and then multiplied by the factor 17 for 9 successive times, it will be employed 14 times as a factor. Therefore $17^5 \times 17^9 = 17^{5+9} = 17^{14}$.

3. What is the quotient of $94^{18} \div 94^{16}$?

Ans. $94^{18-16} = 94^2$.

130. EXAMPLES IN INVOLUTION.

1. Read each of the following numbers: 5^7; 7^5; 3^2; 1.5^3; $.09^6$; 2.063^4.

2. What is the 2d power of 5? the 3d power? the 4th power? the 5th power?

3. What is the square of 11? the cube? the bi-quadrate?

4. What is the square of 2.7? the cube of 40?

5. Find the value of $.2^5$; 64^2; $.12^3$; $.4^4$; $(\frac{1}{2})^6$; $.3^7$.

6. What is the square of 11.7? of $8\frac{3}{7}$?

7. What is the cube of 2.5? of $4\frac{1}{2}$?

8. What is the 4th power of $\frac{2}{3}$? of $2\frac{1}{2}$?

9. What is the difference between 2^8 and 3^2? between 3^7 and 7^3?

10. What is the value of 1^{60}? 2^8? $.08^2$? $3^2 \times 3^3$?

11. What power of 83 is equivalent to $83^7 \times 83^{11}$? to $83^9 \times 83^2 \times 83^{17}$; to $83^{25} \div 83^{20}$? to $83^4 \times 83^{14} \div 83^9$?

12. Divide 31^{17} by 81^{17}; 29^{11} by 29^{10}; 84^3 by 84.

13. Multiply 852^6 by 852^9, and divide the product by 852^{14}.

14. What is the 8th power of 16^3? the 24th power of 196^2?

131. Evolution.

Evolution is the process by which we discover the root of any given power. Thus, 3 is the 2d root of 9, the 3d root of 27, the 5th root of 243, because $9 = 3^2$, $27 = 3^3$, $243 = 3^5$. So the 2d or square root of 49 is 7; the 3d or cube root of 125 is 5; the 4th root of 16 is 2; the 5th root of 1024 is 4, &c. We may denote a root by a *radical sign*, or by a *fractional exponent*.

The radical sign is $\sqrt{}$, and when employed by itself it denotes the square root. If we wish to denote the 3d, 5th, 7th, &c., root, the index of the root is written above the radical sign, thus, $\sqrt[3]{}$, $\sqrt[5]{}$, &c. In fractional exponents, the numerator expresses the power of the number, and the denominator expresses the root. Thus, $(27)^{\frac{1}{3}} = \sqrt[3]{27}$; $(16)^{\frac{3}{4}} = \sqrt[4]{16^3}$; $(32)^{\frac{4}{5}} = \sqrt[5]{32^4}$, &c.

TABLE OF ROOTS AND POWERS.

1st power,	1	2	3	4	5	6	7	8	9
2d power,	1	4	9	16	25	36	49	64	81
3d power,	1	8	27	64	125	216	343	512	729
4th power,	1	16	81	256	625	1296	2401	4096	6561
5th power,	1	32	243	1024	3125	7776	16807	32768	59049
6th power,	1	64	729	4096	15625	46656	117649	262144	531441
7th power,	1	128	2187	16384	78125	279936	823543	2097152	4782969
8th power,	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th power,	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
10th power,	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
11th power,	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
12th power,	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481
13th power,	1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	2541865828329
14th power,	1	16384	4782969	268435456	6103515625	78364164096	678223072849	4398046511104	22876792454961
15th power,	1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649

132. EXERCISES ON THE TABLE OF ROOTS AND POWERS.

1. What is the 5th power of 2? the 5th root of 32?

2. What is the 7th power of 3? the 7th root of 2187?

3. What is the 9th power of 5? the 9th root of 1953125?

4. What is the 3d power of 9? the 3d root of 729?

5. What is the 8th power of 4? the 8th root of 65536?

6. What is the 10th power of 6? the 10th root of 60466176?

7. What is the 15th power of 3? the 15th root of 14348907?

133. EXTRACTION OF THE SQUARE ROOT.

The square root of a number is *the number which, when multiplied by itself, will produce the given number.*

In the following table are the numbers from 1 to 10, inclusive, and beneath them are their squares; therefore, the numbers of the second line have for their square roots the numbers of the first.

Roots	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Squares	1,	4,	9,	16,	25,	36,	49,	64,	81,	100

If there are decimals in the root, there will be twice as many in the square; because any product contains as many decimals as both factors. And conversely, there will be half as many decimals in the root as in the square.

Every entire number, which is not the square of another entire number, is an imperfect second power. For the root of such a number cannot be expressed by a fraction, because a fraction multiplied by itself would give a fractional product. It must therefore be a surd.

The least square of units, is . . . . $1 \times 1 = 1$
The least square of tens, is . . . . $10 \times 10 = 100$
The least square of hundreds, is . . . $100 \times 100 = 10000$
The greatest square of units, is . . . $9 \times 9 = 81$
The greatest square of tens and units, is $99 \times 99 = 9801$
The greatest square of hundreds, tens, and units, is . . . . $999 \times 999 = 998001$

Hence we may see, that if we divide a square number into periods of two figures, by placing a point over units, and one over each second figure to the left, the number of periods will denote the number of figures in the root. Thus, the square root of $\dot{1}4\dot{4}$ contains two figures, and is 12. The square root of $1\dot{6}0\dot{0}$ contains 2 figures, and is 40.

134. EXAMPLE FOR THE BOARD.

What is the square root of 1369?

$$\begin{array}{r} 37 \\ 37 \\ \hline 49 \\ 21 \\ 21 \\ 9 \\ \hline 1369 \end{array}$$

The square of 30 is 900. The square of 37 is 1369, and to discover in what manner this square is formed, we will multiply 37 by 37, writing each product separately, instead of adding them as we proceed. We then see that we must add the square of the tens, twice the product of the tens by the units, and the square of the units.

Let us now reverse the process, and extract the square root of 1369.

$$\begin{array}{rr} & 1\dot{3}6\dot{9}\,(37 \\ & 9 \\ \cline{2-2} 67) & 469 \\ & 469 \\ \cline{2-2} \end{array}$$

Pointing the periods, we find the root will consist of two figures, and the square of the tens must therefore be contained in the 13 hundreds. The greatest square in 13 is 9, and the root 3 is written as the first figure of the required root. Subtracting the square of the root already found, the remainder, 469, must contain twice the product of the tens by the units + the square of the units. To obtain the units' figure, we divide the 46 tens by $2 \times 3 = 6$ tens, which gives a quotient 7. Writing the 7 in the root, and also at the right of the divisor, we multiply by 7, and obtain 469, which is *twice the product of the tens by the units plus the square of the units.** Hence we deduce the following

RULE:

Separate the number into periods of two figures each, by placing a point over the units' figure, and another over each second figure to the left, (and also to the right, if decimals are desired in the root.) Write in the quotient the root of the greatest square contained in the left-hand period, and subtract its square from the period.

To the remainder annex the second period, and divide the tens of the number thus formed, by twice the first quotient figure, placing the result in the root, and also at the right of the new divisor. Multiply the completed divisor by the new quotient figure, subtract the product from the dividend, and annex the third period to the remainder.

Double the root figures already found for a new trial divisor, and proceed as before, until all the periods are brought down.

[When any trial divisor is not contained in the tens of the dividend, place a zero in the root, and also at the right of the divisor, and bring down the next period.

If any figure obtained for the root proves too large, diminish it by one and repeat the work.

Approximate roots may be obtained by annexing decimal periods of two figures each.]

---

* The formation of the trial divisors will perhaps be more readily understood if explained in the following manner:

After subtracting the square of the tens, the remainder will be 2 × tens × units + $(\text{units})^2$, which is equivalent to (2 × tens + units) × units. Now, if we knew the value of the first of these factors, we could easily obtain the other by dividing. We therefore take the part that we do know, (2 × tens,) for a trial divisor, and, seeing how many times it is contained, we add the quotient to complete the divisor.

135. EXAMPLES IN SQUARE ROOT.

1. Extract the square root of 529; of 961.

2. Extract the square root of 1444.

3. What is the square root of 3249? of 3969?

4. What is the square root of 12321?

5. What is the square root of 2256.25?

6. What is the square root of 1.907161?

7. What is the square root of 53300033424?

8. Find the square root of .0361; of .00040804.

9. Find the square root of $\frac{9}{16}$; $\frac{25}{49}$; $1\frac{7}{9}$; $7\frac{29}{100}$.

10. Find the approximate square root of 2, to 6 decimal places. *Ans.* 1.414213.

11. Find the approximate square root of 63; 1.6; .009; 27.1; 14367.

12. Find the square root of $318\frac{82}{169}$; $7\frac{1}{9}$; $7\frac{9}{16}$; $4\frac{2}{5}$; $17\frac{1}{2}$.

13. How many rods long is the side of a square field that contains 225 acres? *Ans.* 189.73 *rods.*

14. What is the side of a square which contains as much as a circle whose area is 273.5 square feet? *Ans.* 16.538*ft.*

15. An army of 8649 men is arranged in a solid square. How many are there on each side?

16. A certain oblong field contains 40 acres, and the length is 4 times the width. Required the length and breadth.

[N. B. The field can be divided into 4 squares, each of which contains 10 acres. The side of one of these squares is the width of the field.]

17. Extract the square root of 625; of 676.

18. Extract the square root of 1024; of 2025.

19. What is the square root of 1.1449? of 1.092025?

20. What is the square root of 107.1225?

21. Find the approximate square root of 750.

22. Find the approximate square root of 1903.

23. The square root of $\frac{3}{7}$ is how many sevenths?

Ans. $\frac{\sqrt{21}}{7}$, or $\frac{4.582+}{7}$.

[The square root of any fraction is equivalent to another fraction, whose numerator is the square root of the product of the numerator by the denominator, and whose denominator is the same as that of the original fraction. It is also equivalent to a fraction whose numerator is the same as that of the original fraction, and whose denominator is the square root of the product of the original numerator by the denominator.

Ex. $\sqrt{\frac{5}{9}} = \frac{\sqrt{5 \times 9}}{9}$ or $\frac{5}{\sqrt{5 \times 9}}$; $\sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$ or $\frac{2}{\sqrt{6}}$.

PROOF. $\sqrt{\frac{5}{9}} = \sqrt{\frac{5}{9}} \times \sqrt{\frac{9}{9}} = \frac{\sqrt{5 \times 9}}{9}$; $\sqrt{\frac{5}{9}} = \sqrt{\frac{5}{9}} \times \sqrt{\frac{5}{5}} = \frac{5}{\sqrt{5 \times 9}}$.

$\sqrt{\frac{2}{3}} = \sqrt{\frac{2}{3}} \times \sqrt{\frac{3}{3}} = \frac{\sqrt{2 \times 3}}{3}$; $\sqrt{\frac{2}{3}} = \sqrt{\frac{2}{3}} \times \sqrt{\frac{2}{2}} = \frac{2}{\sqrt{3 \times 2}}$.]

24. The square root of $\frac{4}{5}$ is how many fifths?

25. The square root of $\frac{3}{11}$ is how many elevenths?

26. The square root of $\frac{5}{8}$ is how many eighths?

27. The square root of $4\frac{2}{3}$ is how many thirds?

28. The square root of $7\frac{1}{2}$ is how many halves?

29. Reduce the square root of $\frac{3}{8}$ to a fraction whose numerator is 3.

30. Reduce the square root of $\frac{2}{5}$ to a fraction whose numerator is 2.

31. Reduce the square root of $\frac{7}{13}$ to a fraction whose numerator is 7.

32. Reduce the square root of $\frac{6}{7}$ to a fraction whose numerator is 6.

33. Find the square root of $3\frac{5}{7}$; of $4\frac{3}{4}$; of $9\frac{1}{2}$.

34. Find the square root of $14\frac{5}{8}$; of $27\frac{4}{9}$; of $69\frac{5}{11}$.

136. EXTRACTION OF THE CUBE ROOT.

The cube root of a number is the *number which, when raised to the third power, will produce the given number.*

In the following Table are the numbers from 1 to 10 inclusive, and beneath them are their cubes; therefore, the numbers of the second line have for their cube roots the numbers of the first.

Roots	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Cubes	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000.

Thus we see that there are only nine integral cubes between 1 and 1000. All the other intermediate integers are imperfect cubes, and their roots can only be obtained approximately.

All perfect cubes from 1 to 1000, evidently have but one integral figure in their cube root. All numbers between 1000 or 10^3, and 1000000 or 100^3, will have two figures in their root. And generally, if we divide a cube into periods of three figures each, by placing a point over units, and one over every third figure from units, the number of points will show the number of figures in the root.

137. EXAMPLES FOR THE BOARD.

In order properly to understand the principles of the cube root, the student should be provided with the following blocks:

1. A cubical block, each side measuring 3 inches, to represent the CUBE OF THE TENS.

2. Three blocks, each 3 inches square and $\frac{7}{10}$ of an inch thick, to represent THE SQUARE OF THE TENS MULTIPLIED BY THE UNITS.

3. Three blocks, each $\frac{7}{10}$ of an inch square and 3 inches long, to represent THE SQUARE OF THE UNITS MULTIPLIED BY THE TENS.

4. A cubical block, each side measuring $\frac{7}{10}$ of an inch, to represent THE CUBE OF THE UNITS.

If block No. 1 be placed upon a table, the three blocks No. 2, laid against three of its sides, the three blocks No. 3, in the deficiencies left by No. 2, and No. 4 in the corner still unfilled, we shall have a new cube, which will represent THE CUBE OF THE TENS, *plus* THREE TIMES THE SQUARE OF THE TENS MULTIPLIED BY THE UNITS, *plus* THREE TIMES THE SQUARE OF THE UNITS MULTIPLIED BY THE TENS, *plus* THE CUBE OF THE UNITS.

This set of blocks is readily applicable in showing the difference between 30^3 and 37^3. But it will be easily perceived that the same result would have been obtained, if, instead of 3 inches, and $\frac{7}{10}$ of an inch, we had employed 4 inches and $\frac{9}{10}$ of an inch, — 11 inches and $\frac{2}{10}$ of an inch,—or any other numbers to represent tens and units. Therefore, the cube of any number whatever is equal to *the cube of the tens,* + *three times the square of the tens* × *the units,* + *three times the tens* × *the square of the units,* + *the cube of the units.*

---

* The formation of the cube may also be illustrated by the method adopted in explaining the square root. See Common School Arithmetic, Chap. XIII., § 181.

We will cube 37, both by our rule and by direct multiplication.

The cube of the tens,	= 27	thous.
3 × the square of the tens × the units,	= 189	hund.
3 × the tens × the square of the units.	= 441	tens,
The cube of the units,	= 343	units,
The cube of 37	= 50653	

$$\begin{array}{r} 37 \\ 37 \\ \hline 37^2 = 1369 \\ 37 \\ \hline 37^3 = 50653 \end{array}$$

Let it now be required to extract the cube root of 50653.

		$5\dot{0}65\dot{3}(37$
		27
	tr. div.	
$3 \times 3^2 =$	27.49	23653
$3 \times 30 \times 7 =$	6.30	23653
comp. div.	3379	

Dividing the number into periods, we find there will be two figures in the root. 27 is the greatest cube contained in the first period, and its root, 3, is evidently the tens' figure of the root sought. Subtracting 27 from 50, and annexing the next period to the remainder, we have 23653, which must evidently contain 3 × the square of the tens × the units, + 3 × the tens × the square of the units + the cube of the units, which is the same as, the units × (3 × the square of the tens + 3 × the tens × the units + the square of the units). To discover the units' figure, we will employ 3 × the square of the tens, or 27 hundreds, for a trial divisor. 27 hundreds is contained in 236 hundreds 8 times. But if we complete our divisor with 8, it will be found too large; we therefore try 7. 3 × the tens × the units = 63 tens, or 630. The square of the units is 49 units, which is annexed to the 27 hundreds. Adding these numbers, we obtain 3379 for our complete divisor, which, being multiplied by 7, gives 23653,—the remaining part of the root. Hence we derive the following

RULE:

Separate the number into periods of three figures, by placing a point over the units' figure, and one over each third figure to the left, (and also to the right, if decimals are desired in the root.) Write in the quotient the root *of the greatest cube contained in the left-hand period, and subtract the cube from the period, annexing the second period to the remainder.*

Take three times the square of the root already found, for a trial divisor, and find how many times this divisor is contained in the hundreds of the dividend. Place the result in the quotient, and its square at the right of the trial divisor, (supplying the place of tens with a zero, if the square is less than ten.)

Multiply 30 *times the root already found, by the last root figure. This product added to the divisor, will give the complete divisor.*

Multiply the complete divisor by the last root figure, subtract the product from the dividend, annex the next period to the remainder, and proceed as before until the whole root is extracted.

If any complete divisor is not contained in its dividend, a zero must be annexed to the root, and two to the trial divisor, and the next period brought down for a new dividend.

If any figure obtained for the root is too great, diminish it by 1 *or more, and repeat the work.*

[It may be well to illustrate to the pupil the algebraical formation of the cube, by raising 30 + 7 to the third power.

In forming the complete divisor, we say, "Multiply 30 times the root already formed, by the last root figure," because 3 × the tens × the units would give tens, and a zero must be annexed for units.

The square of the units is written at the right of the trial divisor, to render the work more compact. This square being units, and the trial divisor hundreds, the two may very properly be united.]

What is the cube root of 226802438.843904?

$$
\begin{array}{lr}
 & 22\dot{6}80\dot{2}43\dot{8}.84\dot{3}90\dot{4}(609.84 \\
 & 6^3 = 216 \\
 & \overline{} \\
3 \times 6^2 \overset{\text{tr. div.}}{=} 108{,}01 & 108{,}02 \\
30 \times 6 \times 1 = 180 & \\
\overline{10981} \text{ c. div.} & \\
\text{which is not contained in the dividend.} & \overline{} \\
3 \times 60^2 \overset{\text{tr. div.}}{=} 10800{,}81 & 108024{,}38 \\
30 \times 60 \times 9 = 16200 & \\
\overline{1096281} \text{ com. div.} & 9866529 \\
3 \times 609^2 \overset{\text{tr. div.}}{=} 1112643{,}64 & \overline{9359098{,}43} \\
30 \times 609 \times 8 = 146160 & \\
\overline{111410524} \text{ com. div.} & 891284192 \\
3 \times 6098^2 \overset{\text{tr. div.}}{=} 111556812{,}16 & \overline{446256519{,}04} \\
30 \times 6098 \times 4 = 731760 & \\
\overline{11156412976} \text{ com. div.} & \underline{44625651904}
\end{array}
$$

The trial divisors, after the first, may be more conveniently found, *by adding to the last complete divisor the last number used to complete it, and twice the square of the last root figure.* Thus in the foregoing example, our third trial divisor =

$$
\begin{array}{rr}
 & 1096281 \\
 & 16200 \\
2 \times 9^2 = & 162 \\
\text{trial divisor,} & \overline{1112643}
\end{array}
$$

138. EXAMPLES IN CUBE ROOT.

1. Extract the cube root of 1331; of 1728; of 2197.
2. Extract the cube root of 2744; of 4096; of 12167.
3. Extract the cube root of 29791; of 8000; of 6859.
4. Extract the cube root of 5832; of 4913; of 10648.

5. Extract the cube root of 24389; of 21952; of 27000.

6. Extract the cube root of 64000; of 125000; of 9261.

7. Extract the cube root of 3375; of 13824; of 15625.

8. What is the cube root of 328509? *Ans.* 69.

9. What is the cube root of 2248091? *Ans.* 131.

10. What is the cube root of 9129329? *Ans.* 209.

11. What is the cube root of 345.357380096? *Ans.* 7.016.

12. What is the cube root of .062570773? *Ans.* .397.

13. What is the cube root of $\frac{8}{27}$; $\frac{125}{729}$; $\frac{512}{1000}$?

14. What is the cube root of $5\frac{13}{343}$; $11\frac{25}{64}$; $274\frac{5}{8}$?

15. What is the cube root of $\frac{148867}{1030301}$? *Ans.* $\frac{53}{101}$.

16. What is the cube root of 4.160075243787? *Ans.* 1.6083.

17. What is the side of a cubical box that will hold 1 bushel? *Ans.* 12.91*in.*

18. What is the side of a cubical pile that contains 258 cords of wood? *Ans.* 32.08*ft.*

19. What is the cube root of 6; $\frac{1}{2}$; $2\frac{1}{3}$; 95; 8.71?
Ans. 1.817; .794; 1.326; 4.563; 2.057.

20. What is the cube root of .0081? *Ans.* .2008.

21. The cube root of $\frac{2}{3}$ is how many thirds?

Ans. $\frac{\sqrt[3]{18}}{3}$, or $\frac{2.62+}{3}$.

[The cube root of any fraction $\frac{a}{b}$ is equal to $\frac{a}{\sqrt[3]{a^2b}}$ or to $\frac{\sqrt[3]{ab^2}}{b}$.]

Ex. $\sqrt[3]{\frac{2}{4}}=\frac{2}{\sqrt[3]{4\times 4}}$, or $\frac{\sqrt[3]{2\times 16}}{4}$; $\sqrt[3]{\frac{3}{5}}=\frac{3}{\sqrt[3]{45}}$ or $\frac{\sqrt[3]{75}}{5}$.

Proof. $\sqrt[3]{\frac{2}{4}} = \sqrt[3]{\frac{2}{4}} \times \sqrt[3]{\frac{2 \times 2}{2 \times 2}} = \frac{2}{\sqrt{4 \times 4}}.\ \sqrt[3]{\frac{2}{4}} = \sqrt[3]{\frac{2}{4}} \times \sqrt{\frac{4 \times 4}{4 \times 4}} = \frac{\sqrt[3]{2 \times 16}}{4}.$

22. The cube root of $\frac{3}{4}$ is how many fourths?

23. The cube root of $\frac{2}{5}$ is how many fifths?

24. The cube root of $\frac{4}{7}$ is how many sevenths?

25. The cube root of $\frac{1}{6}$ is how many sixths?

26. The cube root of $\frac{7}{8}$ is how many eighths?

27. What is the cube root of $\frac{8}{27}$? of $\frac{1}{64}$? of $42\frac{7}{8}$?

28. Find the cube root of $\frac{1}{1000}$; of $13\frac{103}{125}$; of $91\frac{1}{8}$.

29. Find the cube root of $21\frac{119}{125}$; of $857\frac{3}{8}$; of $74\frac{11}{125}$.

30. Find the cube root of $\frac{125}{216}$; of $18\frac{26}{27}$; of $\frac{125}{729}$.

31. Find the cube root of $20\frac{51}{64}$; of $\frac{8}{27000}$; of $166\frac{3}{8}$.

32. Find the cube root of $296\frac{8}{27}$; of $\frac{729}{8000}$; of $1\frac{91}{125}$.

33. What is the cube root of $81\frac{10}{27}$? of $1157\frac{5}{8}$? of 475.647962616?

XVIII. PROGRESSION, OR SERIES.

139. Arithmetical Progression.

A series of numbers, in which the successive terms increase or diminish uniformly by the same number, is called an Arithmetical Progression,—Progression by Difference, or Equidifferent Series. The difference between the successive terms, is called the *common difference.* The first and last terms of the series are called the *extremes;* the other terms the *means.*

Thus, in the ascending series,—

2, 4, 6, 8, 10, 12, 14, 16,

the extremes are 2 and 16, and the common difference is 2.

In the descending series,—

20, 16, 12, 8, 4, 0,

the extremes are 20 and 0, and the common difference is 4.

Any three of the five following things being given, the other two may be found:

1. The first term.
2. The last term.
3. The number of terms.
4. The common difference.
5. The sum of all the terms.

PROBLEM I.

One of the extremes, the common difference, and the number of terms being given, to find the other extreme and the sum of all the terms.

What is the tenth term of an ascending series, the first term being 1 and the common difference 4?

The second term will evidently be 1 + 4; the 3d, 1 + 2 × 4; the 4th, 1 + 3 × 4, and so on to the 10th, which is 1 + 9 × 4 or 37.

What is the sum of the first ten terms of the above series?

To obtain a rule for finding the sum, we will invert the whole series, and write it under itself. In this manner we shall evidently obtain twice the sum of the series. And we may moreover observe, that *the sum of the extremes is equal to the sum of any two terms equally distant from the extremes, or twice the middle term, when the number of terms is odd.*

1,	5,	9,	13,	17,	21,	25,	29,	33,	37.
37,	33,	29,	25,	21,	17,	13,	9,	5,	1.
38,	38,	38,	38,	38,	38,	38,	38,	38,	38.

We thus have 10 times 38, or *the number of terms multiplied by the sum of the extremes,* which is equal to twice the sum desired. Hence we derive the following

RULE:

To obtain the last term: *Multiply the common difference by the number of terms less one, and if the series is ascending, add the product to the first term,—if descending, subtract it from the first term.*

To obtain the sum of the terms: *Multiply the number of terms by the sum of the extremes, and divide the product by two.**

1. The first term of an equidifferent series is 5, the common difference 2, and the number of terms 11. Find the last term, and the sum of all the terms.

2. What is the sum of 45 terms of the natural series, 1, 2, 3, &c.?

3. Find the 23d term, and the sum of 23 terms of the series, 1, 4, 7, 10, &c.

4. How many times does a clock strike in 12 hours?

5. The first term of a descending series is 30, the common difference $\frac{1}{2}$, and the number of terms 33. Required the last term, and the sum of the series.

6. If I travel 100 miles to-day, and 5 miles less on each succeeding day, how far shall I have gone at the end of the 17th day, and what will be the length of the last day's journey?

PROBLEM II.

The extremes and number of terms being given, to find the common difference.

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* If the given term is the *last term,* invert the series, and work by the same rule.

The first term of an equidifferent series is 27, the last term 3, and the number of terms 9. What is the common difference?

The last term is found by Problem I. by subtracting 8 times the common difference from the first term. Then the difference of the extremes 24, must be 8 times the common difference, which is therefore 3.

RULE.

Divide the difference of the extremes by the number of terms less one, and the quotient will be the common difference. This difference repeatedly added to the less, or subtracted from the greater term, will give the intermediate terms.

The sum of the terms is found by the preceding problem.

7. One hundred and fifty eggs were laid in a row. The first was 2 yards, and the last 100 yards from a basket. How far were the eggs apart, and how far must a person travel to pick them up singly and lay them in the basket?

8. A man had 15 children, whose ages were in Arithmetical Progression: the youngest was 3 years old, and the oldest 24. Required the common difference of their ages.

9. The extremes of an equidifferent series are 1 and 11, and the number of terms 25. What are the mean terms?

10. Insert 12 arithmetical means between 20 and 59. [As there are 12 means, the number of terms must be 14.]

11. Insert 13 arithmetical means between 3 and $4\frac{1}{2}$.

12. A man commenced walking for exercise, daily increasing the distance by equal additions. The first day he walked 4 miles, and on the 15th day he walked $7\frac{1}{2}$ miles. What was the daily increase?

PROBLEM III.

The extremes and common difference being given, to find the number of terms.

The extremes are 45 and 10, and the common difference is 5. What is the number of terms?

By Problem I. the difference of the extremes is equal to the common difference multiplied by the number of terms less one. Then the difference of the extremes 35, divided by the common difference 5, gives 7, which must be equal to the number of terms less 1. Therefore the number of terms is 8.

RULE.

Divide the difference of the extremes by the common difference, and add 1 *to the quotient.*

13. What is the sum of the series 2, 4, 6, 8, &c., to 1000?

14. What is the sum of the descending series 100, 99½, 99, 98½, &c., the last term being 0?

15. The first term of a series is 11, the last term 2, and the common difference $\frac{1}{7}$. What is the number of terms?

16. The first term 2½, the common difference $\frac{1}{8}$, and the last term 90, being given, required the number of terms.

PROBLEM IV.

The extremes and sum of the terms being given, to find the number of terms.

RULE.

Divide twice the sum of the terms by the sum of the extremes.

17. The sum of the terms is 22½, the first term 4, and the last term 11. Required the number of terms and the common difference.

PROBLEM V.

One of the extremes, the number of terms, and the sum of the terms being given, to find the other extreme.

RULE.

Divide twice the sum of the terms by the number of the terms, and subtract the given extreme from the quotient.

18. The sum of the terms in an equidifferent series is 27, one of the extremes is 4, and the number of terms 8. Required the other extreme and the common difference.

PROBLEM VI.

The number of terms, common difference, and sum of the terms being given, to find the extremes.

RULE.

Divide the sum of the terms by the number of terms. Then multiply the common difference by the number of terms less one, and subtract half the product from the first quotient for one extreme. For the other extreme, add half the product to the quotient.

Problems IV., V., and VI., are all readily deduced from Problem I.

19. The number of tens in a descending arithmetical series, is 9, the common difference $2\frac{1}{2}$, and the sum of the terms 135. Required the first and the last term.

140. GEOMETRICAL PROGRESSION.

A series of numbers whose successive terms increase or diminish uniformly in the same ratio, is called a PROGRESSION BY QUOTIENT, or GEOMETRICAL PROGRESSION. The numbers may also be regarded as a series of CONTINUAL PROPORTIONALS. The *ratio* has already been defined in the Article on Proportion. As in Arithmetical Progression, the first and last terms are called the *extremes*, the other terms the *means*.

Thus in the ascending progression,—

1, 3, 9, 27, 81, 243,

the extremes are 1 and 243, and the ratio is 3.

In the descending progression,—

24, 12, 6, 3, $\frac{3}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{3}{16}$,

the extremes are 24 and $\frac{3}{16}$, and the ratio is $\frac{1}{2}$.

From the nature of the series, it is evident that any four successive terms of a Geometrical Progression, constitute a proportion; as 1 : 3 : : 9 : 27, 3 : 9 : : 27 : 81, &c., in the first of the above series; and 12 : 6 : : 3 : $\frac{3}{2}$, 6 : 3 : : $\frac{3}{2}$: $\frac{3}{4}$, &c., in the second series.

PROBLEM I.

One of the extremes, the ratio, and the number of terms being given, to find the other extreme.

The first term of an increasing geometrical series is 2, and the ratio 3. What is the sixth term?

The second term will be 2×3; the third, $2 \times 3 \times 3$, or 2×3^2; the fourth, $2 \times 3^2 \times 3$, or 2×3^3, and so on to the sixth, which is 2×3^5. If the sixth term had been given, and the first required, we should evidently have been obliged to divide by 3^5.

RULE.

Raise the ratio to a power whose index is equal to the number of terms less one. Then for the last term multiply, and for the first term divide, the given extreme by this power of the ratio.

We have already seen in Involution, that by adding the exponents of two powers of the same number, we obtain the exponent of their product. Thus $3^7 \times 3^4 = 3^{11}$; $5^2 \times 5^3 = 5^5$, &c. This principle will greatly assist us in finding the powers of the ratio.

What is the 17th power of 2?

$\overset{1}{2}, \overset{2}{4}, \overset{3}{8}, \overset{4}{16}.$ $2^4 \times 2^4 = 256 = 2^8.$ $2^8 \times 2^8 = 256 \times 256 = 65536 = 2^{16}.$ $2^{16} \times 2 = 131072 = 2^{17}.$

In this instance, we form the powers as high as the 4th power, then multiply the 4th power by itself for the 8th power, the 8th power by itself for the 16th power, and the 16th power by the 1st power for the 17th power. We have, therefore, after obtaining the 4th power, only *three* multiplications to make, instead of *thirteen*, which would otherwise have been necessary.

1. The first term of a geometrical series is 3, the ratio 2, and the number of terms 9. What is the last term?

2. What is the 11th term of the series 4096, 1024, 256, &c.?

3. What is the 8th term of a series, whose first term is 20, and ratio 1.06? *Ans.* 30.0726051798272.

4.. What is the amount of $20 for 7 years, at 6 per cent. compound interest?

5. What is the amount of $300 for 9 years, at 6 per cent. compound interest?

6. If the first pane of glass in a window cost 1 mill, the second 2 mills, the 3d 4 mills, the 4th 8 mills, &c., what would be the price of the 12th pane?

7. If $100 were at simple interest for 3 years, at 6 per cent., and the amount then placed at compound interest at 5 per cent., what would be the whole amount at the expiration of 12 years? *Ans.* $183.06.

8. The first term is 8, the ratio $\frac{1}{8}$, and the number of terms 8. What is the last term?

9. The twelfth term is 59049, and the ratio $\frac{1}{3}$. What is the first term? *Ans.* 10460353203.

10. What principal will amount to $4489.643, in 12 years, at 5 per cent. compound interest? *Ans.* $2500.

11. A farmer inquiring the price of a drove of 30 oxen,

was told that he might have the whole drove for the price of the 20th ox, valuing the first at one cent, the second at 2 cents, the third at 4 cents, and so on, doubling the price of each ox for the price of the next. What would be the price of the drove per head at this rate? *Ans.* $174.76.

PROBLEM II.

The extremes and ratio being given, to find the sum of the terms.

The first term of a series is 162, the last term is 2, and the ratio $\frac{1}{3}$. What is the sum of the series?

The series is 162, 54, 18, 6, 2, the sum of which is 242.

54, 18, 6, 2, $\frac{2}{3}$, is another series obtained by multiplying each term of the first series by the ratio. Subtracting the second series from the first, we have $161\frac{1}{3}$, which is the difference between the first term and the last term multiplied by the ratio. It is also $\frac{2}{3}$ of the sum of the first series, since it is obtained by subtracting $\frac{1}{3}$ of the series from the whole series. Then if we divide by $\frac{2}{3}$, or the difference between the ratio and 1, we shall obtain the desired sum.

RULE.

Multiply the last term by the ratio, and divide the difference between the product and the first term by the difference between the ratio and 1.

12. The first term 9, the ratio $\frac{1}{3}$, and the last term $\frac{1}{243}$, are given. What is the sum of the series? *Ans.* $13\frac{121}{243}$.

13. What is the sum of 12 terms of the progression 2, 8, 32, &c.? (The last term is found by Problem I.)

14. What is the sum of the series 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, &c., to infinity? (The last term in any infinite decreasing series is 0.) *Ans.* 4.

15. Required the sum of the infinite series 7, $\frac{7}{3}$, $\frac{7}{9}$, &c.

16. If I lay up \$100 every year, to what will the whole amount in 10 years, at 6 per cent. compound interest? (The 1st term is 100, the ratio 1.06, and the number of terms 10.) *Ans.* \$1318.08.

17. What would be the amount of an annual saving of \$300 for 30 years, at 6 per cent. compound interest?

18. A blacksmith agreed to shoe a horse for the amount of 32 nails, at 1 mill for the first nail, 2 mills for the second, 4 for the third, and so on. What was his charge?

19. What is the sum of the infinite series 6, $1\frac{1}{2}$, $\frac{3}{8}$, $\frac{3}{32}$, &c.? *Ans.* 8.

20. If a man commences at 21 years of age, and annually puts \$500 at compound interest, how much will he be worth when he is 50 years old? *Ans.* \$36819.90.

PROBLEM III.

The extremes and ratio being given, to find the number of terms.

RULE.

Divide the last term by the first, and add one to the index of that power of the ratio, which is equivalent to the quotient.

21. A man made several payments in Geometrical Progression, each being twice as large as the preceding. The first payment was \$4, and the last \$512. What was the number of payments, and what was the whole amount of the debt?

PROBLEM IV.

The extremes and number of terms being given, to find the ratio.

RULE.

Divide the last term by the first, and extract the root of the quotient which is indicated by the number of terms less one.

This and the preceding rule, are readily deduced from Problem I.

22. The first term in a Geometrical series is 1, and the eleventh term is 1024. What is the ratio?

23. The first term is 256, and the fifth term 16. What is the ratio?

24. The first term is 2, the last term $\frac{64}{3125}$, and the number of terms 6. What is the ratio?

25. An estate of $200 amounted to $263.19 in seven years at compound interest. What was the rate per cent.?

26. At what rate per cent. will $100 amount to $1689.48, in nine years at compound interest?

27. Insert 2 mean proportionals between 1 and 6. (As there are to be 2 means, the number of terms is 4, and the extremes 1 and 6.) *Ans.* 1.817; 3.302.

28. Insert 4 mean proportionals between 3 and 96.

29. Insert 5 mean proportionals between 4 and 2916.

30. Insert 3 mean proportionals between 5 and 6480.

The remaining Problems in Geometrical Progression, are of little interest.

XIX. ANNUITIES.

141. ANY sum of money to be paid regularly, at stated periods, is called an ANNUITY. The payment may be stipulated for a given number of years, in which case it is called an *annuity certain,* or it may be dependent upon some particular circumstance, as the life of one or more individuals. The latter is called a *contingent annuity.* A *perpetual annuity,* is one which can only be terminated by the grantor, on the payment of a sum whose interest will be equivalent to the annuity. Of this character is the consolidated debt of England.

An *annuity in possession*, is one on which there is a present claim; an *annuity in reversion*, or deferred annuity, is one that does not commence until the lapse of a stated time, or the occurrence of some uncertain event, as the death of an individual.

The *present worth* of an annuity is the sum which, at compound interest for the time of its duration, would amount to the sum of all the payments, each being placed at compound interest as it became due.

142. PROBLEMS IN ANNUITIES.

PROBLEM I.

To find the amount due on an annuity which has remained unpaid a given time.

As the payments are all at compound interest, this case evidently falls under Problem II., in Geometrical Progression.

RULE.

Find the sum of a Geometrical Progression, in which the first term is the annuity, the ratio is the amount of $1.00 *for the time that should elapse from one payment to another, and the number of terms is the number of payments due.*

PROBLEM II.

To find the present worth of an annuity certain.

What is the present worth of a semi-annual rent of $500 for 6 years, at 6 per cent. ?

By the preceding Problem, we find that the rent at the expiration of the six years, will have amounted to $7096.015. The question is, therefore, to find the present worth of $7096.015 due in 6 years, at a compound interest of 3 per cent. semi-annually, which is done by Problem I., in Geometrical Progression.

RULE.

Divide the amount of the annuity, (found by Problem I.,) by the amount of $1.00 *for the same time.*

PROBLEM III.

To find the present worth of a perpetual annuity.

The present worth of a perpetual annuity, is a sum which would yield an interest equivalent to the annuity. As the interest is found by multiplying the principal by the rate, the principal may be found by dividing the interest by the rate.

RULE.

Divide the annuity by the rate per cent.

PROBLEM IV.

The present worth of a certain annuity being given, to find the annuity.

RULE.

Find the first term of a Geometrical Progression, in which the sum is the amount of the annuity for the whole time, the ratio is the amount of $1.00 *for the time that elapses between two successive payments, and the number of terms is the number of payments.*

PROBLEM V.

To find the present worth of an annuity in reversion.

RULE.

Find the present worth of the annuity until the commencement of the reversion, and also the present worth until its termination. The difference of these two values will give the present worth of the reversion.

The solution of annuity questions by the rules is so tedious, that Tables have been computed, to abridge the labor.

THE AMOUNT OF AN ANNUITY OF $1.00 FROM 1 YEAR TO 50.

Year.	3 p. cent.	3½ p. cent.	4 p. cent.	4½ p. cent.	5 p. cent.	5½ p. cent.	6 p. cent.
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.030000	2.035000	2.040000	2.045000	2.050000	2.055000	2.060000
3	3.090900	3.106225	3.121600	3.137025	3.152500	3.168025	3.183600
4	4.183627	4.214943	4.246464	4.278191	4.310125	4.342266	4.374616
5	5.309136	5.362466	5.416322	5.470710	5.525631	5.581091	5.637093
6	6.468410	6.550152	6.632975	6.716892	6.801913	6.888051	6.975319
7	7.662462	7.779408	7.898294	8.019152	8.142008	8.266894	8.393838
8	8.892336	9.051687	9.214226	9.380014	9.549109	9.721573	9.897468
9	10.159106	10.368496	10.582795	10.802114	11.026564	11.256259	11.491316
10	11.463879	11.731393	12.006107	12.288210	12.577893	12.875354	13.180795
11	12.807796	13.141992	13.486351	13.841179	14.206787	14.583498	14.971643
12	14.192029	14.601962	15.025805	15.464032	15.917127	16.385590	16.869941
13	15.617790	16.113030	16.626838	17.159913	17.712983	18.286798	18.882138
14	17.086324	17.676986	18.291911	18.932109	19.598632	20.292572	21.015006
15	18.598914	19.295681	20.023588	20.784054	21.578564	22.408663	23.275971
16	20.156881	20.971030	21.824531	22.719337	23.657492	24.641139	25.672528
17	21.761588	22.705016	23.697512	24.741707	25.840366	26.996402	28.212880
18	23.414436	24.499691	25.645413	26.855084	28.132385	29.481205	30.905653
19	25.116868	26.357180	27.671229	29.063562	30.539004	32.102671	33.759992
20	26.870374	28.279682	29.778078	31.371423	33.065954	34.868318	36.785591
21	28.676486	30.269471	31.969202	33.783137	35.719252	37.786075	39.992727
22	30.536780	32.328902	34.247970	36.303378	38.505214	40.864309	43.392290
23	32.452884	34.460414	36.617888	38.937030	41.430475	44.111846	46.995828
24	34.426470	36.666528	39.082604	41.689196	44.501999	47.537998	50.815577
25	36.459264	38.949857	41.645908	44.565210	47.727099	51.152588	54.864512
26	38.553042	41.313102	44.311745	47.570645	51.113454	54.965979	59.156383
27	40.709634	43.759060	47.084214	50.711324	54.669126	58.989109	63.705766
28	42.930923	46.290627	49.967583	53.993333	58.402583	63.233510	68.528112
29	45.218850	48.910799	52.966286	57.423033	62.322712	67.711353	73.639798
30	47.575416	51.622677	56.084938	61.007070	66.438847	72.435478	79.058180
31	50.002678	54.429471	59.328335	64.752388	70.760790	77.419429	84.801677
32	52.502759	57.334502	62.701469	68.666245	75.298829	82.677498	90.889778
33	55.077841	60.341210	66.209527	72.756226	80.063770	88.224760	97.343165
34	57.730177	63.453152	69.857909	77.030256	85.066959	94.077122	104.183755
35	60.462082	66.674013	73.652225	81.496618	90.320307	100.251363	111.434780
36	63.275944	70.007603	77.598314	86.163966	95.836323	106.765188	119.120867
37	66.174223	73.457869	81.702246	91.041344	101.628139	113.637274	127.268119
38	69.159449	77.028895	85.970336	96.138205	107.709546	120.887324	135.904206
39	72.234233	80.724906	90.409150	101.464424	114.095023	128.536127	145.058458
40	75.401260	84.550278	95.025516	107.030323	120.799774	136.605614	154.761966
41	78.663296	88.509537	99.826536	112.846688	127.839763	145.118923	165.047684
42	82.023196	92.607371	104.819598	118.924789	135.231751	154.100464	175.950545
43	85.483892	96.848629	110.012382	125.276404	142.993339	163.575989	187.507577
44	89.048409	101.238331	115.412877	131.913842	151.143006	173.572669	199.758032
45	92.719861	105.781673	121.029392	138.849965	159.700156	184.119165	212.743514
46	96.501457	110.484031	126.870568	146.098214	168.685164	195.245720	226.508125
47	100.396501	115.350973	132.945390	153.672633	178.119422	206.984234	241.098612
48	104.408396	120.388257	139.263206	161.587902	188.025393	219.368367	256.564529
49	108.540648	125.601846	145.833734	169.859357	198.426663	232.433627	272.958401
50	112.796867	130.997910	152.667084	178.503028	209.347996	246.217477	290.335906

THE PRESENT WORTH OF AN ANNUITY OF $1.00 FROM 1 YEAR TO 50.

Year.	3 p. cent.	3½ p. cent.	4 p. cent.	4½ p. cent.	5 p. cent.	5½ p. cent.	6 p. cent.	Year.
1	0.97087	0.96618	0.96154	0.95694	0.95238	0.94786	0 94339	1
2	1.91347	1.89969	1.88609	1.87267	1.85941	1.84631	1.83339	2
3	2.82861	2.80164	2.77509	2.74896	2.72325	2.69793	2.67301	3
4	3.71710	3.67308	3.62990	3.58753	3.54595	3.50514	3.46511	4
5	4.57971	4.51505	4.45182	4.38998	4.32948	4.27028	4.21236	5
6	5.41719	5.32855	5.24214	5.15787	5.07569	4.99553	4.91732	6
7	6.23028	6.11454	6.00205	5.89270	5.78637	5.68297	5.58238	7
8	7.01969	6.87396	6.73274	6.59589	6.46321	6.33457	6.20979	8
9	7.78611	7.60769	7.43533	7.26879	7.10782	6.95220	6.80169	9
10	8.53020	8.31661	8.11090	7.91272	7.72173	7.53762	7.36009	10
11	9.25262	9.00155	8.76048	8.52892	8.30641	8.09254	7.88687	11
12	9.95400	9.66333	9.38507	9.11858	8.86325	8.61852	8.38384	12
13	10.63495	10.30274	9.68565	9.68285	9.39357	9.11708	8.85268	13
14	11.29607	10.92052	10.56312	10.22283	9.89864	9.58965	9.29498	14
15	11.93794	11.51741	11.11839	10.73955	10.37966	10.03759	9.71225	15
16	12.56110	12.09412	11.65230	11.23401	10.83777	10.46216	10.10589	16
17	13.16612	12.65132	12.16567	11.70719	11.27407	10.86461	10.47726	17
18	13.75351	13.18968	12.65930	12.15999	11.68959	11.24607	10.82760	18
19	14.32380	13.70984	13.13394	12.59329	12.08532	11.60765	11.15812	19
20	14.87747	14.21240	13.59033	13.00794	12.46221	11.95034	11.46992	20
21	15.41502	14.69797	14.02916	13.40472	12.82115	12.27524	11.76408	21
22	15.93692	15.16712	14.45112	13.78442	13.16300	12.58317	12.04158	22
23	16.44361	15.62041	14.85684	14.14777	13.48857	12.87504	12.30338	23
24	16.93554	16.05837	15.24696	14.49548	13.79864	13.15170	12.55036	24
25	17.41315	16.48151	15.62208	14.82821	14.09394	13.41391	12.78336	25
26	17.87684	16.89035	15.98277	15.14661	14.37518	13.66250	13.00317	26
27	18.32703	17.28536	16.32959	15.45130	14.64303	13.89810	13.21053	27
28	18.76411	17.66702	16.66306	15.74287	14.89813	14.12142	13.40616	28
29	19.18845	18.03577	16.98371	16.02189	15.14107	14.33310	13.59072	29
30	19.60044	18.39205	17.29203	16.28889	15.37245	14.53375	13.76483	30
31	20.00043	18.73628	17.58849	16.54439	15.59281	14.72393	13.92909	31
32	20.38877	19.06887	17.87355	16.78889	15.80268	14.90420	14.08404	32
33	20.76579	19.39021	18.14765	17.02286	16.00255	15.07507	14.23023	33
34	21.13184	19.70068	18.41120	17.24676	16.19290	15.23703	14.36814	34
35	21.48722	20.00066	18.66461	17.46101	16.37419	15.39055	14.49825	35
36	21.83225	20.29049	18.90828	17.66604	16.54685	15.53607	14.62099	36
37	22.16724	20.57053	19.14258	17.86224	16.71129	15.67400	14.73678	37
38	22.49246	20.84109	19.36786	18.04999	16.86789	15.80474	14.84602	38
39	22.80822	21.10250	19.58448	18.22965	17.01704	15.92866	14.94907	39
40	23.11477	21.35507	19.79277	18.40158	17.15909	16.04612	15.04630	40
41	23.41240	21.59910	19.99305	18.56611	17.29437	16.15746	15.13802	41
42	23.70136	21.83488	20.18563	18.72355	17.42321	16.26299	15.22454	42
43	23.98190	22.06269	20.37079	18.87421	17.54591	16.36303	15.30617	43
44	24.25427	22.28279	20.54884	19.01838	17.66277	16.45785	15.38318	44
45	24.51871	22.49545	20.72004	19.15635	17.77407	16.54772	15.45583	45
46	24.77545	22.70092	20.88465	19.28837	17.88007	16.63283	15.52437	46
47	25.02471	22.89943	21.04294	19.41471	17.98102	16.71357	15.58903	47
48	25.26671	23.09124	21.19513	19.53561	18.07716	19.79011	15.65003	48
49	25.50160	23.27656	21.34147	19.65130	18.16872	16.86266	15.70757	49
50	25.72976	23.45562	21.48218	19.76201	18.25593	16.93143	15.76186	50

143. EXAMPLES FOR THE PUPIL.

1. If a person saves $250 per annum, and invests it at 7 per cent. compound interest, how much will he be worth at the end of 25 years? *Ans.* $14530.65.

2. What is the amount of an annuity of $500 payable semi-annually, forborne for 7 years, at 6 per cent. per annum?

3. What is the amount of a quarterly annuity of $400, in arrears for 5 years, at 5 per cent.? *Ans.* $9025.19.

4. What is the amount of an annual salary of $1000 for 10 years, at 6 per cent.?

5. What is the amount of a quarterly rent of $200 for 8 years, at 6 per cent.? *Ans.* $2608.24.

6. What is the amount of a pension of $400 a year, payable semi-annually, for 8 years and 6 months, at 7 per cent. per annum?

7. What is the amount of a rent of $1500, ten years forborne, at 5 per cent. per year?

8. An estate that yields an annual income of $2000, is offered for sale for the amount of 10 years' income at 6 per cent. compound interest. What is the price of the estate?

9. What is the amount of a salary of $1300 for 16 years, at 5 per cent. compound interest? *Ans.* $30754.73.

10. What is the present worth of an annual salary of $800 to continue 8 years, at 5 per cent. compound interest? *Ans.* $5170.57.

11. What is the present worth of an annual income of $500, to continue 11 years, at 6 per cent. compound interest?

12. What is the present worth of a rent of $200, to continue 5 years, at 6 per cent. compound interest?

13. What is the present worth of a semi-annual pension of $175, to continue 9 years, at 7 per cent. compound interest? *Ans.* $2308.19.

14. What is the present worth of an annuity of $3000 for 7 years, at 3 per cent. compound interest?

15. What sum invested at 6 per cent. compound interest, will yield me an income of $1600 per annum, for 25 years?

16. What sum at 5 per cent. will yield an annual income of $1200 for 15 years?

17. A gentleman wishes to present his estate to his children, reserving enough to yield $700 per annum for 15 years. How much must he reserve, allowing 5 per cent. compound interest? *Ans.* $7265.76.

18. For how much should an estate that rents for $175 per year, be sold, to allow the purchaser 6 per cent. interest on his investment?

19. What is the par value of an annual income of £500 in the 4 per cent. consols?*

20. What sum of money must be laid out in the 3 per cent. consols., at 68 per cent. of the par value, to yield an income of £1000? *Ans.* £22666 13*s.* 4*d.*

21. What sum of money at 4½ per cent. will yield an annual interest of $620?

22. What sum, invested in an estate that rents for $400 per annum, will yield an interest of 8 per cent.?

23. A farm rents for $750 per annum. For what price should it be sold, when money is worth 6 per cent. a year?

24. What sum of money must a man lay up annually, to amount to $10000 in 20 years, the investments being all made at 6 per cent. compound interest? *Ans.* $271.84.

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* CONSOLS., is an abbreviation for the consolidated annuities of the British National Debt.

25. The executors of an estate wish to dispose of an unexpired lease that has 8 years to run, for a premium of $1500. What amount must be added to the annual rent, for that purpose? *Ans.* $241.56.

26. What is the present worth of an annuity of $1100, to commence in 3 years and continue for 8 years, interest at 6 per cent.? *Ans.* $5735.25.

27. What is the present worth of a perpetual annuity of $300, to commence in 2 years, at 5 per cent. interest?

28. A father leaves an annual rent of $400 to his eldest child for 5 years, and the reversion of it for the 8 succeeding years to his youngest child. What is the present worth of each legacy, at 7 per cent.?

XX. MISCELLANEOUS PROBLEMS.

144. MENSURATION.

PROBLEM I.

To find the area of any surface bounded by four sides, the opposite sides being equal.

RULE.

Multiply one of the sides by the perpendicular let fall upon it, from the opposite side.

1. The length of an oblong rectangular field is 40 rods, and the breadth 16 rods. How many square rods does it contain? How many acres?

2. What are the contents of a four-sided field, whose opposite sides are equal, the length being 81 rods, and the distance between the longest sides, 13 rods?

3. The average length of Pennsylvania is about 300 miles, and the average breadth 157 miles. What is the area?

4. What is the area of a rhombus, the side being 73.5 rods, and the breadth 61.25 rods?

5. How many square feet in a board that is $14\frac{3}{4}$ft. long, and 11 inches wide?

6. How many square yards in a floor 14.3ft. long, and $10\frac{1}{3}$ft. wide?

PROBLEM II.

To find the area of a trapezoid, or figure of four sides, two of which are parallel.

RULE.

Multiply the sum of the two parallel sides by half the distance between them.

7. The two parallel sides of a trapezoid measure 11 and 15 inches respectively, and the height is 8 inches. What is the area?

8. What is the area of a field, two sides of which are parallel, and measure 72.5, and 89.25 rods, the distance between them being 39 rods?

9. What is the area of a trapezoid, one of the parallel sides measuring 96 rods, the other $63\frac{3}{4}$ rods, and the distance between them being 84.6 rods?

PROBLEM III.

To find the area of a triangle.

RULE.

Multiply one of the sides by one half of the perpendicular let fall from the opposite angle.

10. How many acres in a triangular meadow, one side measuring $127\frac{3}{5}$ rods, and the perpendicular, 82.41 rods?

11. The base of a triangular lot is $96\frac{5}{8}$ rods, and the perpendicular distance from the opposite angle is 35 rods. What is the area?

12. What is the area of a triangle whose base is $11\frac{3}{7}$ inches, and perpendicular, 28.49 inches?

Any surface bounded by straight lines, may be divided into triangles, and the area of each triangle obtained. The sum of the several areas is the area of the whole surface.

PROBLEM IV.

To find the area of a circle.

RULE.

Multiply half the diameter by half the circumference, or multiply the square of the diameter by .7854.

13. The diameter of a circle is $36\frac{2}{5}$ feet. What is the area?

14. What is the area of a circle whose diameter is $9\frac{7}{8}$ miles?

PROBLEM V.

To find the area of an ellipse, the two diameters being given.

RULE.

Multiply the longer by the shorter diameter, and the product by .7854.

15. A house lot in the form of an ellipse has one diameter 110 feet, and the other 45 feet. What is the area?

16. What is the area of an ellipse, whose diameters are 25 and 17.5 feet?

PROBLEM VI.

To find the surface of a sphere.

RULE.

Multiply the square of the diameter by 3.1416; *or, multiply the diameter by the circumference.*

17. What is the area of the earth's surface?

18. The circumference of a globe is 252 inches. What is the area?

19. The diameter of a globe is 11.5 inches. What is the area?

PROBLEM VII.

To find the area of the convex surface of a cylinder.

RULE.

Multiply the circumference of the base by the height of the cylinder.

20. The diameter of a cylindrical column is 6 feet, and the height 60 feet. What is the area of the convex surface?

21. What is the area of the whole surface of a cylinder, whose diameter is 7.5 feet, and height 49 feet?

PROBLEM VIII.

To find the solid contents of a cylinder.

RULE.

Multiply the area of the base by the height.

22. The diameter of a cylinder is 13 inches, and the height 69 inches. What are the solid contents?

PROBLEM IX.

To find the solid contents of any cylindrical body, whose sides taper uniformly,* as the trunk of a tree.

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* Such a body is called the *frustum of a cone.*

RULE.

Multiply together the diameters of the two extremities, and to the product add one-third of the square of the difference of the diameters. Multiply this sum by .7854, *and the product will be the mean area between the two extremities. The mean area multiplied by the length, will give the solid contents.*

23. What are the solid contents of a stick of timber, whose length is 50 feet, the diameter of the larger end 36 inches, and the diameter of the smaller end 30 inches?

24. What are the solid contents of a ship's mast, whose length is 35 feet, the diameter at the base 24 inches, and the smaller diameter 18 inches?

PROBLEM X.

To find the solid contents of a sphere.

RULE.

Multiply the cube of the diameter by .5236.

25. The diameter of a globe is $4\frac{2}{5}$ feet. What are the solid contents?

26. What are the solid contents of the earth, and what does it weigh, supposing the mean weight to be 5 times that of water?

PROBLEM XI.

To gauge, or find the dimensions of a cask.

RULE.

Find the diameter at the bung, the diameter at the head, and the length of the cask, all in inches. Subtract the head diameter from the bung diameter, and note the difference.

If the staves of the cask be much curved, multiply the dif-

ference by .7; if little curved, by .6; if a medium curve, by .65; and if nearly or quite straight, by .55, and add the product to the head diameter. The sum will be a mean diameter, by which the cask is reduced to a cylinder.

Multiply the square of the mean diameter by the length, and divide the product by 359 *for the contents in beer gallons, or by* 294 *for the contents in wine gallons.*

27. How many wine gallons will fill a cask whose bung diameter is 40 inches, the head diameter 30 inches, and the length 50 inches?

28. How many beer gallons will a cask contain, which measures 31 inches at the head, 33 inches at the bung, and 47 inches in length?

29. What are the contents in wine measure of a tub, whose inner diameter at the bottom is 29 inches, at the top 36 inches, and the height 30 inches? (The tub is a *frustum of a cone,* and the solid contents are found by Problem IX.)

30. How many gallons will fill a churn, that is 18 inches in diameter at the bottom, 12 inches at the top, and 3 feet in height?

PROBLEM XII.

To find the carpenters' tonnage of a vessel.

RULE.

Multiply the breadth at the main beam, half the breadth, and the length, together. Divide the product by 95, *and the quotient is the tonnage.*

This is probably the best general rule for forming estimates; but no rule can be given that will produce a perfectly accurate result. The rule employed by government, in the collection of revenue, gives about $\frac{3}{5}$ of the tonnage thus obtained.

31. What is the tonnage of a vessel, whose length is 70 feet, and breadth 25 feet?

32. The length of a vessel is 163 feet, and the breadth 31 feet. Required the tonnage.

33. Find the tonnage of a vessel that is 113½ feet long, and 24¾ feet wide.

34. What is the tonnage of a ship that is 116 feet long, and 31 feet wide?

145. Philosophical Problems.

PROBLEM I.

To find the specific gravity of a body.

RULE.

If the body is heavier than water, weigh it both in water and out of water, and the difference will be the weight lost in the water. Then, the weight lost in the water : the whole weight : : the specific gravity of water : the specific gravity of the body.*

But if the body is lighter than water, attach to it another body heavier than water, so that the two may sink together. Weigh the two together, and the heavier by itself, both in water and in the air, and find the loss of each in the water. Subtract the less loss from the greater, and say, the last remainder : the weight of the body in air : : the specific gravity of water : the specific gravity of the body.

1: A piece of gold weighed $36\frac{4}{7}$dwt. in water, and $38\frac{6}{7}$dwt. in the air. What was the specific gravity?

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* The specific gravity of water is 1. A cubic foot of water weighs about 1000oz., Av. Therefore, the specific gravity of any body in thousandths, will represent the weight of a cubic foot in ounces.

2. What is the specific gravity of a body that weighs 13½ pounds in the air, and 9¾ pounds in water?

3. What is the weight of a block of oak, that contains 13⅖ cubic feet, the specific gravity being .925?

PROBLEM II.

To find the distance at which bodies may be seen at sea, or on level ground, their height and the height of the eye being known.

RULE.

To the earth's diameter, (41815224 *feet,*) *add the height of the eye, and multiply the sum by the height of the eye. The square root of the product is the distance at which an object* ON THE SURFACE *of the earth or water can be seen.*

Work in the same way with the height of the object, and the sum of the two results is the distance at which the object may be seen.

How far may a mountain, that is 1½ miles high, be seen from the masthead of a ship, 50 feet above the surface of the water?

$$\begin{aligned} \sqrt{(41815224 + 50) \times 50} &= 45724\text{ft., or } \ \ 8\tfrac{2}{3}\text{m.} \\ \sqrt{(41815224 + 7920) \times 7920} &= 575534\text{ft., or } \underline{109\ \text{m.}} \\ & \qquad\qquad \textit{Ans. } \ 117\tfrac{2}{3}\textit{m.} \end{aligned}$$

4. How far can Bunker Hill Monument, which is 282 feet above the level of the sea, be seen from the deck of a vessel, the spectator's eye being 15 feet above the water?

5. How far may a mountain 2⅓ miles high, be seen from the masthead of a vessel, 40 feet above the water?

PROBLEM III.

To determine the distance of a gun, or a thunder-cloud, from seeing the flash, and hearing the report.

RULE.

Multiply the number of seconds that elapse between the flash and the report by 1142, *for the distance in feet.*

6. Four and a half seconds after seeing the flash of a cannon, the report was heard. What was the distance?

7. What is the distance of an electrical cloud, if the thunder is heard in $2\frac{1}{2}$ seconds after the flash is seen?

PROBLEM IV.

To find the pressure of water against the banks of a stream or the dam of a pond.

RULE.

Multiply the area of the bank by one-half the depth of the water, for the cubical contents of a column of water equivalent to the pressure.

8. The gate of a floom is 18 feet deep and 16 feet wide. What pressure does it sustain?

9. What amount of pressure is sustained by a bank whose area is 5694 feet, the average depth of water being 10.5 feet?

146. MISCELLANEOUS EXAMPLES.

1. The tail of the comet of 1811, when at its greatest length, was computed to extend 23 million miles. Reduce this sum to inches.

2. The year 45 B. C., the year in which the Julian calendar was introduced, was made to contain 445 days. How many more seconds did it contain than a solar year?

3. Reduce to miles and decimals of a mile, the height of each of the following volcanoes: Etna, 10870ft.; Vesuvius, 3932ft.; Hecla, 5500ft.; Cotopaxi, 18867ft.; Antisana, 19136ft.

4. Eaton Hall, in England, the seat of the Earl of Grosvenor, cost £400000.* If this amount should be invested at 6 per cent., how many teachers could be supported from the income, at an average salary of £150?

5. Plymouth Breakwater, which contains 2000000 tons of stone, was built at the estimated expense of £1170000.* Estimating the pound sterling at $4.84, what was the cost of the stone per ton?

6. How many pounds of lead could be extracted from 175 tons of galena, the ore yielding 70 per cent.?

7. The Vatican, at Rome, contains 11000 apartments.* How many acres does it cover, the length being 1200ft., and the breadth 1000ft.?

8. The cathedral of Milan is 454ft. long, 270ft. wide, and the height of the tower is 335ft.* Find the dimensions of a model, 4ft. long.

9. The head of the Scheremetor family, in Russia, is said to have 125000 serfs, valued at 150 roubles each.* What would be their value in Federal Money, at 40 cents per rouble?

10. The cathedral church of Kasan, in St. Petersburg, cost fifteen million roubles, and was built after a design furnished by a slave.* Reduce the cost to French currency, estimating 9 roubles as equivalent to 20 francs.

11. A mass of silver was detached by lightning from a mountain near La Paz, in Bolivia, for which $11269 was paid.* What would be the weight of such a mass, supposing it to have been sold at $1.16 per ounce Avoirdupois?

12. The walls of Media, 100ft. high and 20ft. thick, reached 75 miles from the Tigris to one of the canals of the Euphrates.† Find their contents in solid yards.

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* Murray.

† Edinburgh Review.

13. The Taj Mahul, in India, the mausoleum of Shah Jehan's queen, who died in 1631, occupied 20000 men for 22 years, and cost £31748021.* Estimating the average wages of each man at $3 a week, what would be the cost of the materials used in building it?

14. The coal fields of America have been estimated to cover 7000 square miles. If the average yield is 6000000 tons per square mile, and the annual consumption 12000000 tons, how long will it be before the supply is exhausted?

15. The canal connecting Amsterdam with the Helder, is 50 miles long, 125ft. wide at the surface, 36ft. at the bottom, and 21ft. deep.† How many hogsheads would it contain? *Ans.* 52992000*hhd.*

16. The colossal equestrian statue of Peter the Great, stands on a block of black marble that weighs 1500 tons.† If the specific gravity of the marble is 2600,‡ what are its solid contents? *Ans.* 20677*c. ft. nearly.*

17. Find the cube root of 9, to six decimal places.

18. By accounts accurately kept from 1730 to 1752, the number of deaths in Batavia, in an average population of 70000, amounted to 1100000.† What was the annual percentage of deaths?

19. The salt springs of Salina, in New York, yield a bushel of salt for every 45 gallons of water. In 1826, they yielded 827508bu.; in 1830, 1485446bu.; in 1835, 2209867bu.† How many hogsheads of the water were evaporated in each of these three years?

20. The flea, locust, and grasshopper, jump 200 times their own length. If men could do the same, what would be the height of a man who could jump $\frac{1}{5}$ of a mile?

21. It has been calculated by many naturalists, that the

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* Edinburgh Review. † Murray. ‡ 2600oz. per cubic foot.

elm-tree produces yearly 630000 seeds. If every seed should be fertile, what area would be covered by the produce of a single crop, allowing each tree to occupy 100 square feet?

22. It has been calculated that an acre of land, containing 640 trees, of the average diameter of eight inches, will throw off, through the leaves, 3875 gallons of water in 12 hours. At this rate, what will be the average exudation of each tree, per hour?

23. The largest known current of lava, is the one which, in 1783, covered a territory in Iceland, 20 leagues in length and 4 in breadth.* Estimating the average depth at 3 feet, and the weight at 1 ton per cubic yard, what would be the weight of the whole?

24. Powerscourt Fall, in Wicklow county, Ireland, is 360 feet high.* At what distance is an observer, who finds that a two-foot rule, held 18 inches from the eye, intercepts the rays from the top and base of the Fall?

25. The most elevated of the Pyrenees, is Mount Perdu, which is 11168ft. high. How would this altitude be represented, assuming the height of Mount Etna, (10870ft.,) as the unit?

26. Not less than $1647901018 were obtained from the silver mines of Potosi, in 255 years.* What was the average produce per annum, in sterling money, estimating the pound at $4.84?

27. The cathedral of Montreal, which is 255ft. long, 134ft. wide, and 220ft. high, will hold 10000 people.* What would be the dimensions of a building on the same model, that would hold 5000?

Partial Ans. 155$\frac{1}{2}$*ft. high.*

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* Murray.

28. Broadway, in the city of New York, is 80ft. wide, and 2 miles long.* How many acres does it contain?
Ans. $19\frac{13}{33}$ *acres.*

29. According to Herodotus, the walls of Babylon were 60 miles in circumference, 365ft. high, and so broad that 6 chariots could drive abreast. Estimating the average breadth of the space occupied by each chariot, at 5ft., how many cubic yards would such a wall contain, if solid, the city being square? *Ans.* $128431333\frac{1}{3}$ *yards.*

30. On one occasion, in the year 1760, a pile of aromatics was burned, near the Stadt-house in Amsterdam, valued at 8 million livres.* Estimating the pound sterling at $25\frac{3}{4}$ livres, or $4.80, find the value of the pile in Federal Money. *Ans.* $1491262.13.

31. The Fall of Tequendama, near Bogota, is 40ft. wide, and 650ft. high.* Reduce each of these dimensions to decimals of a furlong.

32. Gmelin asserts that, at Tobolsk, a man may live comfortably on $1\frac{1}{2}$ guineas per annum. At this rate, what population could have been supported by the army expenditures of the United States, in 1847, ($41281606,) estimating the English shilling at 24 cents? *Ans.* 5460529.

33. The fishery of coral, in Algiers, produces an annual value of about £1000000. How many laborers could be hired with this amount, at 24s. 7d. per week?
Ans. 15645 *laborers.*

34. Find the 8th root of 3512479453921.

35. The distance from Washington to Portland is 542 miles.† If a man should start from Washington, and travel 3 miles the first day, 5 miles the second day, and so on, in

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* Murray.

† U. S. Almanac.

arithmetical progression, in what time would he reach Portland? *Ans.* $22\frac{7}{24}$ *days.*

36. If the multiplicand is 7, and the product 2, what is the multiplier?

37. The dividend is 1, and the quotient 8; what is the divisor?

38. The dividend is $6\frac{1}{2}$, and the quotient $18\frac{2}{3}$; what is the divisor?

39. The sum of two numbers is $7\frac{1}{3}$, and one of the numbers is 4.759; what is the other?

40. The difference of two numbers is $13\frac{3}{7}$, and the greater number is 29.43; what is the less?

41. The solid contents of one of the blocks of marble, in the walls of Baalbec, is 9072c. ft.* What would be the side of a cubical block, of the same solidity?

42. On the Sparrowhill, in Moscow, the Emperor Alexander, in 1817, laid the foundation of the "Temple of our Saviour," which, if completed, will be the most gigantic structure in the known world. It is to be 770ft. high, fronted by a colonnade 2000ft. long.† Required the least number of miles which is a common multiple of these dimensions. *Ans.* 175 *miles.*

43. K. has a farm of 400 acres, for which he gave $75 per acre. There is a bed of sand extending over the whole, which is worth $1 per ton, the expense of digging and delivering being 50ct. per ton. Allowing 6 per cent. interest on the cost of the land, and estimating the yield of sand at 5 tons per square rod, what is his profit on the sale of 10000 tons per annum?

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* Nesbit.

† Murray.

44. The sum of two numbers is $7\frac{5}{19}$, and their difference is $6\frac{3}{31}$; what are the numbers?

45. What number must be subtracted from $39\frac{5}{9}$ to leave $17\frac{2}{25}$?

46. What number must be added to $23\frac{5}{11}$ to make 47.432?

47. What number must be multiplied by $3\frac{7}{8}$, and the product divided by 7.365, to give $8\frac{3}{4}$ as a quotient?

48. What is the difference between $\frac{3}{46}$ and $\frac{4.6}{3}$ of 7T. 15cwt. 3qr.?

49. In the tomb of one of the Incas, £150000 in gold was found.* What was the weight of the whole, estimating the average fineness of the gold at 18 carats, a pound sterling containing 113.001 grains of pure gold?

Ans. 1*T.* 8*cwt.* 3*qr.* 8.6*lb.*

50. It has been estimated that the loss of life from accidents, in every mile of travelling by steam, is only as 1 to 14661467, and the reception of any injury whatever, is only as 1 to 7850738.† At this rate, how many times could a steamer, with 50 passengers, cross the Atlantic, (3000 miles,) without accident, and how many times without loss of life? And what would be each passenger's chance of safety?

Partial Ans. 52 *times without accident;* 97 *times without loss of life.*

51. What is the difference between 28 miles, and 27m. 7fur. 39r. 5yd. 2ft. 11in.?

52. Reduce $\frac{1}{7}$ of 9m. 7fur. 39r. 5yd. 2ft. to inches.

53. If from a purse containing £35 7s. 11d., I pay to each of 15 laborers, £1 9s. $8\frac{3}{4}$d., how much will be left?

54. Find the sum, the difference, and the product of 874.91, and $42\frac{7}{13}$.

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* Murray.

† Lardner.

55. The density of the atmosphere is supposed to diminish in a geometrical ratio, being only $\frac{1}{2}$ as great at an elevation of 3 miles, as at the surface of the earth. The density at the surface being represented by 1, what will represent the density at the height of 12 miles? At the height of 18 miles? 30 miles? 15 miles? 24 miles?

56. There are 48 libraries in Europe, numbering 100000 volumes and upwards,* viz:

LIBRARY.	VOLS.	AUTHORITY.
Paris National Library,	824000	Brit. Quart. Rev., VI., 113.
Munich Royal Library,	600000	De Reiffenberg, Pélerinage.
St. Petersburg Imp. Lib.,	446000	Russ. Off. Reports.
London British Museum,	435000	Parl. Returns, 1849.
Copenhagen Royal Library,	412000	Constantin Bib. Pub.
Berlin Royal Library	410000	Edwards' App. to Rep.
Göttingen University,	360000	Do. do.
Vienna Imperial,	313000	Serapeum, Nov., 1846.
Dresden Royal,	300000	Petzholdt, Dresden, Bib.
Breslau University,	250000	Edwards, ut supra.
Do. do.	320000	Petzholdt Addressbuch.
Oxford Bodleian,	220000	Parl. Returns, 1849.
Madrid National,	200000	Rep. on B. Museum, 1836.
Tubingen University,	200000	Edwards' App. to Rep.
Munich do.	200000	Do. do.
Wolfenbüttel Ducal,	200000	Petzholdt Anzeiger.
Heidelberg University,	200000	Meyer. Convers. Lex.
Stuttgard Royal,	187000	Petzholdt Addressbuch.
Paris Arsenal,	180000	Br. Quart. Rev., VI., 103.
Milan Bresa,	170000	Eland., North Italy.
Cambridge University,	166724	Parl. Returns, 1849.
Hamburg City,	152000	Edwards' App. to Rep.
Darmstadt G. Ducal,	150000	Petzholdt Addressbuch.
Paris St. Genevieve,	150000	Rep. on B. Museum.
Florence Magliabecchian,	150000	Guida di Firenze.
Naples Royal,	150000	Valery Voy. Liv., XII. c. v.
Bologna University,	150000	Edwards, ut supra.

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* J. G. Cogswell, Esq.

LIBRARY.	VOLS.	AUTHORITY.
Gotha G. Ducal,	150000	Petzholdt Addressbuch.
Edinburgh Advocates',	148000	Parl. Returns, 1849.
Ajaccio Town,	140000	Dict. Ency. de la France.
Brussels Royal,	133500	Annuaire de la Biblioth.
Prague University,	130000	Edwards, ut supra.
Rome Casanate,	120000	Constantin Bib. Pub.
Vienna University,	115000	Serapeum, Nov., 1846.
Leipsic do.	112000	Petzholdt Addressbuch.
Copenhagen do.	110000	Report on B. Museum.
Turin do.	110000	Edwards' App. to Rep.
Weimar G. Ducal,	110000	Petzholdt Addressbuch.
Bordeaux city,	110000	Dict. Ency. de la France.
Louvain University,	105000	Edwards, ut supra.
Dublin T. College,	104239	Do.
Rome, Vatican,	100000	
Hague, Royal,	100000	Rep. on Br. Museum.
Paris Mazarine,	100000	Brit. Quar. Rev., VI., 113.
Parma Ducal,	100000	Constantin Bib. Pub.
Erlengen University,	100000	Petzholdt Addressbuch.
Upsala University,	100000	Baird's Northern Europe.
Escurial Royal Library,	100000	Greatly overrated.
Bonn University,	100000	Petzholdt Addressbuch.

Assuming the library at the University of Turin as the unit, what numbers would represent each of the others?

57. From $\frac{5}{7}$ of 3T. 17cwt., subtract $\frac{4}{13}$ of 7T. 3cwt. 1qr. 18lb.

58. What will be the freight of 17$\frac{3}{5}$cwt. for 89$\frac{5}{7}$ miles, if $7.63 be paid for carrying 11$\frac{3}{13}$ tons, 9$\frac{1}{3}$ miles?

59. Bought 16cwt. 3qr. 16lb. of rice, at $4.00 per cwt., and 9cwt. 2qr. 5lb. of pearl barley, at 4.37\frac{1}{2}$ per cwt. How much would be gained on the whole, by selling each at 4$\frac{3}{4}$ cents a pound?

60. If 63$\frac{5}{8}$yd. of broadcloth cost $255, at what price must it be sold per yd. in order to gain $25.50?

61. A hogshead of sugar at $7.00 per cwt. cost $43.75. What did it weigh?

62. Smith & Jones commence business with a capital of $5000, and at the expiration of 4 years, their profits have been so great that their capital has increased to $18500, the amount of their sales, during the fourth year, being $35700. Estimating the ratio of the sales to the capital, as uniform, it is required to find the average annual percentage of increase in the amount of business, the amount of business transacted each year, and the average percentage of the profits on the amount of business.

Ans. Average annual percentage of increase 38.69 + *per cent.;*
Amount of business, 1st year, $13381.88;
2d year, $18559.55;
3d year, $25740.55;
4th year, $35700;
Percentage of profits on the amount of business, 14.456 + *per cent.*

63. Kensington Gardens, in London, contain 227 acres; Hyde Park, 380A.; Green Park, 56A.; St. James's Park, connected with Green Park, 87A.; Regent's Park, 372A.; terraces and canals connected with Regent's Park, 80A.; public squares in various parts of London, 100A.[*] To how many enclosures of the size of Boston Common, which contains 45 acres, are these grounds equivalent?

64. The atmosphere has been estimated to contain 3000 billion lb. of carbon. How many cords of wood would weigh the same, at 700lb. per cord?

65. How much shalloon that is $\frac{3}{4}$yd. wide will line $14\frac{5}{8}$ yards of cloth that is $1\frac{3}{4}$yd. wide?

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[*] Colman.

66. What is the price of 16 boxes of raisins, each holding $9\frac{2}{5}$lb., at $9\frac{1}{4}$ cents per lb.?

67. How much money that is 9 per cent. below par, will pay a debt of $187.50?

68. If 9 men mow 10 acres of grass in a day, how much will 11 men mow in $2\frac{1}{2}$ days?

69. In what time will $150 gain 6.37\frac{1}{2}$ at 6 per cent. simple interest?

70. Dr. Ure estimates the latent heat of steam at 967°. At this estimate, how much coal would be sufficient to raise the temperature of a hogshead of water 10°, supposing 8.4lb. to sufficient to convert a c. ft. of boiling water into steam? *Ans.* $\frac{11319}{15472}$*lb.*

71. "A traveller pays away one-half at Pryag; from the remainder he pays two-ninths at Benares; from the remainder he pays one-fourth for customs; from the remainder six-tenths are paid at Gaya, and there remain 63, which he brings home. What was the original sum?"*

72. If $41.\dot{6}$ per cent. of the profits of a business are allowed for salaries, and the balance is divided among the partners in proportion to their capital, and if the gross profits of a year's sales are $37\frac{1}{2}$ per cent. of the capital, what percentage does the capital gain? *Ans.* 21.9 *per cent.*

73. The length of the Capitol at Washington, is 350 feet, and the height of the dome is 145ft. Find in sevenths, the square root of the ratio of the height of the dome to the length of the building. *Ans.* $\frac{1{\cdot}702938+}{7}$.

74. When molasses is $31\frac{1}{4}$ cents a gallon, how many hogsheads, each holding 97gal. 3qt., can I buy for 366.56\frac{1}{4}$?

75. What will be the price of 7 bales of sheeting, each

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* Lilawati.

bale containing 9 pieces, and each piece measuring $30\frac{3}{4}$yd., if 26yd. cost \$$2.92\frac{1}{2}$?

76. Bought $39\frac{1}{2}$ bushels of potatoes for \$$12.87\frac{1}{2}$. At what price per bushel must they be sold, in order to gain 15 per cent.?

77. What is the interest of \$9431 for 3yr. 7mo. 13dy. at 7 per cent.?

78. How much may a man spend per day, whose income is \$500 a year, after deducting $\frac{5}{10}$ per cent. for taxes?

79. According to Chateauvieux, the population of France was estimated, at the time of his writing, at 32000000, of which the land proprietors and their families embraced 20000000; the agricultural laborers and their families, 3000000; and the artisans employed in agricultural districts, 2000000; making 25000000 residents of agricultural districts, and leaving only 7000000 who dwelt in towns.* Required the percentage of the entire population represented by each of these classes.

80. By the same authority it appears, that there are, in France, about 4000000 landed proprietors, of whom 2787112 are small proprietors, occupying an average of twelve English acres each; and of this number 1243200 are possessed of various quantities, none exceeding five English acres.* Find the percentage of the entire population represented by each of these numbers.

81. The anthracite coal region of Pennsylvania, covers 624000 acres.† To how many square miles is this equivalent?

82. A silver mine in Pavillon, Mexico, yielded \$4000000 in 6 months.† At this rate, what would be the annual yield in pounds sterling, estimating the pound at \$4.84?

Ans. £1652892 11*s.* 3*d.*

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* Dr. Swift, "Education and Crime."

† Murray.

83. How many pieces of merino that is worth \$8$\frac{5}{8}$ per yd., can be bought for \$71.25, there being 28$\frac{1}{2}$ yards in a piece?

84. Bought 7 hogsheads of sugar at \$6.75 per cwt. To what did it amount, the gross weight being 6cwt. 3qr. 19lb., tare 25lb. per hogshead?

85. "The Twenty-first Annual Report of the Treasurer of the Worcester County Institution for Savings, respectfully submitted to the corporation, is as follows:

From the 12th day of April, 1848, to the 11th day of April, 1849, he has received, in 3106 deposits, the sum of		\$171525.94
There have also been added to the accounts of depositors the semi-annual dividends of January and July 1848, amounting to		41030.95
And the extra dividend made at the end of five years, in July, 1848, being 2$\frac{1}{2}$ per cent. per year, in addition to the semi-annual dividends, . .		79059.41
		\$
In the same time he has paid, in 2534 payments to depositors, the sum of		\$302741.65
The reduction of the capital, or amount standing to the credit of the depositors since April 1848, is		
Number of new accounts opened, .	1,106	
Number of accounts closed, . .	1,209	
Present number of depositors, . .	6,514	
Average amount of depositors' accounts,		

STATE OF WORCESTER COUNTY INSTITUTION FOR SAVINGS, WEDNESDAY, APRIL 11, 1849.

The Treasurer charges himself for amount due to 6514 depositors, and credited on their accounts,	\$1059124.13
For balance due to sundry depositors, not on interest, ...	1027.20
Dividend of January, 1849, ...	20487.01
Profits received and undivided	3784.69

And is credited—

By amount of Bank Stock, ...	$137830.00
By United States Stock, ...	102700.00
By Notes of the County of Worcester and of Towns,	176819.07
By Notes secured by pledge of Boston Water Loan,	6900.00
By Notes secured by pledge of Bank Stock, .	32600.00
By Notes secured by Mortgage of Real Estate, .	547531.16
By Notes with Personal Security, . . .	75692.44
By Cash, ...	$
	$

April 18, 1849. SAM'L JENNISON, Treasurer."

Supply all the blanks in the foregoing report.

86. What will be the price of 13 bags of cotton, each weighing 5cwt. 1qr. 11lb., at $11\frac{1}{2}$ cents per pound?

87. If $\frac{1}{2}$ of $\frac{3}{4}$ of $5\frac{1}{3}$yd. of cloth cost \$$10\frac{5}{8}$, what will 17.63 yards cost?

88. At £1 11s. per yard, what will be the price of 3qr. 3na. of cloth?

89. The silver mine of Valenciana, in Guanaxuato, yielded 165 million dollars between the years 1766 and 1803.* What was the average yield per month?

90. What is the fifth power of $17\frac{2}{3}$?

91. According to a recent estimate, the annual consumption of lucifer matches in Great Britain is about 5000000000. What would they be worth, at a halfpenny a gross?

92. The thickness of a soap bubble, just before it bursts, is about $\frac{1}{4000000}$ of an inch. How many such thicknesses would reach from the earth to the sun, 96000000 miles?

93. A. lent B. $685 for $3\frac{1}{2}$ months; what sum ought A. to receive in return, to use $7\frac{2}{3}$ months?

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* Murray.

94. How much coffee, at $8\frac{1}{4}$ cents a pound, should be given in exchange for $\frac{3}{11}$ of 2cwt. 2qr. 16lb. of tea, at 75 cents a pound?

95. If \$63.87$\frac{1}{2}$ will purchase $8\frac{2}{5}$cwt. of butter, how much must I give for 7cwt. 1qr. $13\frac{1}{2}$lb.?

96. A bankrupt can pay but $63\frac{1}{3}$ per cent. of his debts. How much will be received by a creditor for \$765.62$\frac{1}{2}$?

97. If $\frac{3}{7}$ of $\frac{5}{12}$ of a ship be sold for \$2150, what is the value of $\frac{2}{3}$ of $\frac{8}{9}$ of the remainder?

98. The fibrils of the strawberry leaf-stalk are not more than $\frac{1}{12000}$ of an inch in diameter. How many could be arranged in a square foot?

99. A bar 1 inch square, of best cast iron, may be extended by a weight of $9\frac{3}{4}$ tons; a bar of the best wrought iron by a weight of 30 tons. What weight would be required to extend a bar of each of these materials, $1\frac{1}{2}$in. in diameter? *Ans.* $21\frac{15}{16}$ *tons;* $67\frac{1}{2}$ *tons.*

100. A bin 6ft. long, 4ft. wide, and 5ft. high, is filled with coal. How much coal is there, allowing a cubic yard for a ton?

101. Some one has made the following calculation:

"I have been married 32 years, during which time I have received from the hands of my wife three cups of coffee each day, of half a pint each; yet in that period, I have scarcely varied in weight myself, from 160lb." How many gallons of coffee did he drink during the time, and how many times his own average weight, estimating the weight of the coffee at 1 lb. 2oz. per pint?

102. What is the value of 87 barrels of oil, each containing $85\frac{7}{8}$gal., at $9\frac{3}{4}$ cents per pint?

103. If a laborer receives \$1.50 for every working day, how much can he spend each year, and save \$125?

104. When taxes on all real and personal property are raised at 3 mills on every dollar, what must be paid by a man who is worth $7650, and who pays for 3 polls at $1.50 each?

105. How much hay will 63 horses eat in $21\frac{1}{3}$ weeks, if $19\frac{3}{5}$cwt. will keep 4 horses $6\frac{1}{2}$ weeks?

106. A garrison of 500 men has provisions for 6 weeks, but finding that they will be obliged to remain 15 weeks without an additional supply, how many men must be sent away, that the provisions may not be exhausted?

107. If the weight of a globe of air 1ft. in diameter, is $\frac{1}{25}$lb. Avoirdupois, and the weight of a similar globe of hydrogen is $\frac{1}{150}$lb., what weight would be sustained by a balloon 12ft. in diameter, filled with hydrogen?

Ans. 57.6*lb.*

108. On the Belgian railroads, in the years 1835 to 1839, 6609215 persons travelled, of whom 15 were killed, and 16 were wounded. What percentage of the entire number were killed, and what percentage were wounded?

109. The distances in the following Table are given in nautical miles.* Reduce each of them to statute miles.

			MILES.		MILES.
From	Persian Gulf	to England,	11360	To Oregon,	10260
"	Bombay	" "	11500	" "	9630
"	Calcutta	" "	12275	" "	8700
"	Shanghae	" "	14400	" "	5000
"	Jesso	" "	15660	" "	3660
"	Singapore	" "	12300	" "	7020
"	Batavia	" "	12000	" "	7450
"	N.W. End N. Guinea	"	14000	" "	5670
"	" " Australia	"	11360	" "	8040
"	N.E. " Australia	"	13560	" "	6120
"	Sidney	"	12800	" "	6570
"	Wellington	"	13600	" "	6120

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* James B. Moore.

110. The annual value of cornelian exported from India once amounted to £11000.* Reduce this amount to Federal Money, estimating £1 = 20.75 francs, and $1 = 5.30 francs.

111. In a single year, the banana will yield 4000lb. of nutritive substance, in a space of 100 square metres.* To what amount per acre is this product equivalent?

112. The Natural Bridge of Virginia is 93ft. long, 80ft. wide, and 225ft. above the water. Find the least common multiple of these three numbers.

113. If the store expenses of a manufacturing business are 10 per cent. of the sales, what percentage of net profit will be realized on the sales, provided the cost of manufacturing is $3\frac{1}{2}$ cents per lb., and the goods are sold at $4\frac{1}{2}$ cents per lb.? *Ans.* $13\frac{47}{81}$ *per cent.*

114. England and Wales contain 57960sq. m.; Scotland, 30500sq. m. Divide 92883 in proportion to these two numbers.

115. The square root of $\frac{2}{3}$ is how many thirds?

116. The total expenditures of the United States, for the year ending July 1, 1849, were $57631667.82.† What would be the interest of this amount for 3yr. 6mo. 19dy., at 5 per cent.? *Ans.* $10237625.44.

117. The debt of the United States, on the 1st of October, 1849, was $64704693.71.† What would be the compound interest of this amount for 13yr. 8mo., at 6 per cent.? *Ans.* $78826177.35.

118. The value of goods, wares, and merchandise, imported into the United States from July 1, 1848, to June 30, 1849, was $147857439; of which $22377665 was free

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* Murray.

† American Almanac, 1851.

of duty. What was the average duty on the balance, the total amount received from customs, during the year, being $28346738.82?* *Ans.* 22.59+ *per cent.*

119. The value of the exports of the growth, produce, and manufacture of the United States, for the year ending June 30, 1849, was $132666955.* What percentage of the imports for the year were the exports?

120. The length of post-routes in the United States, for the year ending June 30, 1849, was 167703 miles; the total transportation of the mails over those routes, 42544069 miles; the total cost of transportation, $2428515.* Find the average number of times that the mail was carried over each mile, and the average cost of transportation per mile.

121. The public lands ceded to the United States by treaty with foreign powers, cover an estimated area of 1584000000 acres; of which 146000000 acres had been sold previous to the 30th of September, 1849.* What population would the unsold lands support, if they were settled as densely as Great Britain, which contained, in 1841, 27019555 inhabitants, on an area of 116700 square miles?* *Ans.* 520219045.

122. By an experiment made for the purpose of testing the comparative value of the red and the white ash coals, the following result was obtained: 31 lb. per day of the Schuylkill red-ash coal gave a mean temperature of 64°, and 37lb. per day of the white-ash, taken from a vein of high repute in the Lehigh region, gave a mean temperature of 63°.† If an additional $\frac{1}{2}$lb. per day, of the white-ash would have raised the average temperature 1°, how many pounds of the white-ash would be equivalent in heating

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* American Almanac, 1851.

† Statistics of Coal, by Richard C. Taylor.

power to 2000lb. of the red-ash, and what would white-ash coal be worth per ton, when red-ash sells at $5.50?

2d Ans. $4.55.

123. What part of 17A. 3R. 29r. are 11A. 2R.?

124. Reduce $16\frac{3}{5}$, 14.7, $\frac{11}{13}$, $5\frac{1}{4}$, and $\frac{29}{52}$ to a common denominator.

125. If a staff 3ft. 3in. high cast a shadow of 2ft. 7in., how long a shadow will be cast at the same time by a steeple that is $59\frac{5}{6}$yd. high?

126. Bought 38 pieces of Denims, each piece containing $29\frac{3}{4}$yd., for $162.50. At what price must I sell $16\frac{1}{4}$yd., in order to gain 17 per cent. on the prime cost?

127. How much cotton, at $16\frac{1}{2}$ cents a pound, can be bought for $$512\frac{1}{8}$?

128. The quantity of anthracite coal mined in Pennsylvania, increased from 365 tons, in 1820, to 3242641 tons, in 1849.* What was the total percentage of increase in the 29 years? *Ans.* 888294.79 *per cent.*

129. Of the passengers arriving in the United States, from September 30, 1848, to September 30, 1849, 179253 were males, 119915 were females, and of 442 the sex is not stated in the returns.† Supposing the proportion to be the same, how many of the last-mentioned number were males? *Ans.* 265.

130. The city reservoir of the Boston Water-works, on Beacon Hill, covers an area of 40000 feet, and holds 3000000 gallons of water. What is its depth?

Ans. 10*ft.*

131. The Brookline reservoir of the Boston Water-

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* Statistics of Coal, by Richard C. Taylor.

† American Almanac.

works covers 30 acres, and has an average depth of 15 feet. How many gallons will it contain?

Ans. 146633143 *gallons.*

132. On the Manchester railroad, at least half a million persons travel annually.* Supposing each person to save one hour in the time of transit between Manchester and Liverpool, how many working days of ten hours each, are saved by that single road?

133. The saving referred to in the preceding example, is equivalent to an addition to the actual power of the country, without increasing the quantity of food consumed, of how many persons? *Ans.* 160, *nearly.*

134. The slide of Alpnach, in Switzerland, was built in 1818, to convey the timber from Mount Pilatus to the Lake of Geneva, a distance of 44000 feet. The large pines, which were about a hundred feet long, ran through this space in $2\frac{1}{2}$ minutes.* What was their average velocity per hour? *Ans.* 200 *miles.*

135. A Georgia merchant bought goods to the amount of $575, and obtained credit till January 31, 1851. But being unable to pay the whole debt at one time, he remitted $100 when it became due, and subsequently paid $75 March 3, $100 March 27, $150 April 17, and the balance June 7. What was the amount of the last payment, the legal rate of interest being 8 per cent.? *Ans.* $158.51.

136. A corn merchant bought wheat at $1.20, at $1.10, at $.90, and at $.70 per bushel; but the markets having fallen, he is desirous to sell at $.80 per bushel, and is willing to lose 20 per cent. In what proportions may a mixture be made, to answer the conditions of the question?

Ans. 3*bu. at* $1.20, 1*bu. at* $1.10, 1*bu. at* $.90, 2*bu. at* $.70.

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* Babbage.

137. A refiner melts 14lb. of gold, 22 carats fine, with 16lb. of 20 carats fine. How much alloy must be added, in order to make the composition 18 carats fine?

Ans. $4\frac{8}{9}$*lb.*

138. In 1840, Prof. Bessel, from the corrected parallax of the star 61 Cygni, estimated its distance from the earth to be such, that its light is at least 3391 days, or about 10 years, in reaching us. What would be its distance, according to this estimate, the velocity of light being 192000 miles per second?

139. If we estimate the size of the stars as nearly uniform, their apparent magnitude must depend on their distance from the eye. On this supposition, how long would the light be in coming to us, from a nebula that is first resolved by the mirror of Lord Rosse's telescope, (like the nebula in Orion, which is visible to the naked eye,) which magnifies 500 times,—supposing that the naked eye can discern stars twelve times as remote as 61 Cygni?*

Ans. 60000 *years.*

140. If any of the nebulæ which are barely visible to Lord Rosse's telescope, resemble, in their relative distance from the resolvable nebulæ, the nebula of Orion, how long would their light be in reaching us?

Ans. 30000000 *years.*†

141. A grocer bought 7cwt. 3qr. 13lb. of coffee for $70.48. What per cent. did he gain by selling the whole at 9 cents a pound?

142. Sold 89cwt. 2qr. 17lb. of coffee for $7.50 per cwt., thereby losing $37.50. What did it cost per pound?

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* Nichol.

† Yet the distance of such a nebula would be less than 200 quintillion miles.

143. In what time will $487.50 at 5 per cent. gain the same interest as $943.25, in 7 months, at 6 per cent.?

144. If 15 men, in 21½ days, mow 801 acres of grass, how much will 84 men mow in 7¾ days?

145. In the manufacturing district of Bethnal Green, England, the average age at death in the year 1839, was as follows:

No. of Deaths.	Occupation.	Average Age of Deceased.
79	Gentlemen and persons engaged in professions, and their families	44
824	Tradesmen, farmers, and their families . .	27
3395	Operatives, laborers, and their families . .	19*

Required the average age of all the above deceased.

Ans. 21 *years, nearly.*

146. The cube root of the 6th power of 174, is how many times the fifth root of the tenth power of 29?

147. For the first five weeks of the year 1851, the gross railway traffic of Great Britain and Ireland amounted to £1129965; for the corresponding period of the year 1850, to £964602. What was the increase per cent.?

Ans. 17.148+ *per cent.*

148. A letter in the Norwich Courier, from a merchant in San Francisco, states that Mr. John M. Hosmer, of San Jose, California, raised the following vegetables in the year 1850, on a ranch of 150 acres:

130 acres of potatoes, yielding 35000bu., at $5 per bu.
4 acres of onions, yielding 40000lb., at 40ct. per lb.
15 acres of cabbages, yielding 70000 heads, of which 40000 were sold at 40ct. per head.
80000lb. pumpkins at 6ct. per lb.
Total . . . $

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* British Sanitary Reports.

Allowing $\frac{1}{2}$ of the proceeds for the expenses of cultivation, what principal, at 6 per cent. would yield a sum equivalent to the balance?

149. How much indigo, at $1.75 per pound, can be bought for £15 9s., when exchange is $9\frac{1}{2}$ per cent. above par?

150. Two pieces of cloth, each containing $31\frac{3}{8}$yd., cost $311, but one being of better quality, cost $17.75 more than the other. What was the price of each per yard?

151. By selling tea at $87\frac{1}{2}$ cents a pound, a grocer gains 22 per cent. What did he give for 1cwt. 1qr. $11\frac{3}{4}$lb.?

152. If 73 men can do a piece of work in 63 days, how many men must be added, to complete it in 15.75 days?

153. Mr. Chadwick, the Secretary of the British Board of Poor-Law Commissioners, estimates the amount annually expended in the United Kingdom, for tobacco and intoxicating drinks, (exclusive of wines,) at £50000000 per annum. If this sum were annually invested, at 5 per cent. compound interest, to what would it amount in 40 years?

154. Of 4629 deaths among the laboring classes in Manchester, England, in the year 1840, there were

2649	under 5 years of age.
215	above 5 and under 10.
107	above 10 and under 15.
135	above 15 and under 20.*

Find the percentage of deaths at each of these periods.

155. A circle, made by Ertel, of Munich, with a diameter of only $3\frac{1}{4}$ft., is divided, at its circumference, into thirtieths of a degree; and each of these subdivisions may be

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* British Sanitary Reports.

again divided, by an accompanying micrometer, into 1200 equal parts.[*] How many subdivisions may thus be made in one inch of the circumference?

156. Bessel supposes that the sun moves with a speed so swift, that we might reach the star 61 Cygni in 41000 years. What is the estimated velocity of the sun per second? [See Ex. 138.] *Ans.* 43.5+ *miles.*

157. Determine, from the following Table,[†] the number of square miles of surface for each mile of coast, in each of the grand divisions of the world.

COUNTRIES.	Surface, in square geographical miles.	Length of shores, in geographical miles.
Europe,	2688000	17200
Asia,	14128000	30800
Africa,	8720000	14000
Australia,	2208000	7600
North America,	5472000	24000
South America,	5136000	13600

158. Estimating the diameter of the earth at 7912 miles, and the height of the highest mountain at 28000 feet, what elevation would represent the mountain, on a globe 10 feet in diameter? *Ans.* $\frac{875}{10879}$ *of an inch.*

159. Lieut. Walsh, U. S. N., on the 15th November, 1849, sunk a lead in the Atlantic Ocean, to the depth of 5700 fathoms, without touching bottom.[†] Reduce this depth to geographical miles.

160. At Mahabaleshwar, in the western Ghauts, south of Bombay, 302 inches of rain have been observed to fall in a single year.[†] To how many hogsheads per week would this be equivalent, on each square mile?

Ans. $1597680\frac{240}{511}$ *hogsheads.*

161. If an engine has sufficient force to draw 100 tons

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[*] Nichol.

[†] Guyot.

over level ground, what additional power must be exerted on an ascending grade of 35ft. per mile?

Ans. 13*cwt.* 1$\frac{1}{33}$*qr.*.

162. Place four 9's in such a way that they will represent 100; 20; 111.

163. The average power of draft of a horse, moving 3 miles per hour for 10 hours a day, being 100lb., what will be the annual cost of transportation over a road 45 miles long, on which the average friction is $\frac{1}{20}$ of the weight, estimating the amount transported at 40000 tons, and the value of a day's labor of a horse at 75 cents?

Ans. $50400.

164. If the road in the preceding example should be improved by macadamizing, or otherwise, so that the friction should be reduced to $\frac{1}{50}$ of the weight, what would be the annual cost of the transportation, and how much might be profitably expended in making the improvement?

2*d Ans.* $504000.

147. Table of Prime and Composite Numbers.

The following Table contains all the prime numbers, and the factors of all *odd* composite numbers, below 21000, the prime numbers being indicated by a dash.

For numbers below 1000, all the factors are given. For the odd numbers above 1000, one or more factors will be found in the Table, which will reduce each number to a prime, or to some number less than 1000, and under the latter number the remaining factors may be found.

The hundreds are placed at the head of the Table, and the tens and units at the left hand.

	0	1	2	3	4	5	6	7	8	9
00	—	$2^2.5^2$	$2^3.5^2$	$2^2.3.5^2$	$2^4.5^2$	$2^2.5^3$	$2^3.3.5^2$	$2^2.5^2.7$	$2^5.5^2$	$2^2.3^2.5^2$
01	—	—	3.67	7.43	—	3.167	—	—	$3^2.89$	17.53
02	—	2.3.17	2.101	2.151	2.3.67	2.251	2.7.43	$2.3^3.13$	2.401	2.11.41
03	—	—	7.29	3.101	13.31	—	$3^2.67$	19.37	11.73	3.7.43
04	2^2	$2^3.13$	$2^2.3.17$	$2^4.19$	$2^2.101$	$2^3.7.9$	$2^2.151$	$2^6.11$	$2^2.3.67$	$2^3.113$
05	—	3.5.7	5.41	5.61	$3^4.5$	5.101	5.11^2	3.5.47	5.7.23	5.181
06	2.3	2.53	2.103	$2.3^2.17$	2.7.29	2.11.23	2.3.101	2.353	2.13.31	2.3.151
07	—	—	$3^2.23$	—	11.37	3.13^2	—	7.101	3.269	—
08	2^3	$2^2.3^3$	$2^4.13$	$2^2.7.11$	$2^3.3.17$	$2^2.127$	$2^5.19$	$2^2.3.59$	$2^3.101$	$2^2.227$
09	3^2	—	11.19	3.103	—	—	3.7.29	—	—	$3^2.101$
10	2.5	2.5.11	2.3.5.7	2.5.31	2.5.41	2.3.5.17	2.5.61	2.5.71	$2.3^4.5$	2.5.7.13
11	—	3.37	—	—	3.137	7.73	13.47	$3^2.79$	—	—
12	$2^2.3$	$2^4.7$	$2^2.53$	$2^3.3.13$	$2^2.103$	2^9	$2^2.3^2.17$	$2^3.89$	$2^2.7.29$	$2^4.3.19$
13	—	—	3.71	—	7.59	$3^3.19$	—	23.31	3.271	11.83
14	2.7	2.3.19	2.107	2.157	$2.3^2.23$	2.257	2.307	2.3.7.17	2.11.37	2.457
15	3.5	5.23	5.43	$3^2.5.7$	5.83	5.103	3.5.41	5.11.13	5.163	3.5.61
16	2^4	$2^2.29$	$2^3.3^3$	$2^2.79$	$2^5.13$	$2^2.3.43$	$2^3.7.11$	$2^2.179$	$2^4.3.17$	$2^2.229$
17	—	$3^2.13$	7.31	—	3.139	11.47	—	3.239	19.43	7.131
18	2.3^2	2.59	2.109	2.3.53	2.11.19	2.7.37	2.3.103	2.359	2.409	$2.3^3.17$
19	—	7.17	3.73	11.29	—	3.173	—	—	$3^2.7.13$	—
20	$2^2.5$	$2^3.3.5$	$2^2.5.11$	$2^6.5$	$2^2.3.5.7$	$2^3.5.13$	$2^2.5.31$	$2^4.3^2.5$	$2^2.5.41$	$2^3.5.23$
21	3.7	11^2	13.17	3.107	—	—	$3^3.23$	7.103	—	3.307
22	2.11	2.61	2.3.37	2.7.23	2.211	$2.3^2.29$	2.311	2.19^2	2.3.137	2.461
23	—	3.41	—	17.19	$3^2.47$	—	7.89	3.241	—	13.71
24	$2^3.3$	$2^2.31$	$2^5.7$	$2^2.3^4$	$2^3.53$	$2^2.131$	$2^4.3.13$	$2^2.181$	$2^3.103$	$2^2.3.7.11$
25	5^2	5^3	$3^2.5^2$	$5^2.13$	$5^2.17$	$3.5^2.7$	5^4	$5^2.29$	$3.5^2.11$	$5^2.37$
26	2.13	$2.3^2.7$	2.113	2.163	2.3.71	2.263	2.313	$2.3.11^2$	2.7.59	2.463
27	3^3	—	—	3.109	7.61	17.31	3.11.19	—	—	$3^2.103$
28	$2^2.7$	2^7	$2^2.3.19$	$2^3.41$	$2^2.107$	$2^4.3.11$	$2^2.157$	$2^3.7.13$	$2^2.3^2.23$	$2^5.29$
29	—	3.43	—	7.47	3.11.13	23^2	17.37	3^6	—	—
30	2.3.5	2.5.13	2.5.23	2.3.5.11	2.5.43	2.5.53	$2.3^2.5.7$	2.5.73	2.5.83	2.3.5.31
31	—	—	3.7.11	—	—	$3^2.59$	—	17.43	3.277	$7^2.19$
32	2^5	$2^2.3.11$	$2^3.29$	$2^2.83$	$2^4.3^3$	$2^2.7.19$	$2^3.79$	$2^2.3.61$	$2^6.13$	$2^2.233$
33	3.11	7.19	—	$3^2.37$	—	13.41	3.211	—	$7^2.17$	3.311
34	2.17	2.67	$2.3^2.13$	2.167	2.7.31	2.3.89	2.317	2.367	2.3.139	2.467
35	5.7	$3^3.5$	5.47	5.67	3.5.29	5.107	5.127	$3.5.7^2$	5.167	5.11.17
36	$2^2.3^2$	$2^3.17$	$2^2.59$	$2^4.3.7$	$2^2.109$	$2^3.67$	$2^2.3.53$	$2^5.23$	$2^2.11.19$	$2^3.3^2.13$
37	—	—	3.79	—	19.23	3.179	$7^2.13$	11.67	$3^3.31$	—
38	2.19	2.3.23	2.7.17	2.13^2	2.3.73	2.269	2.11.29	$2.3^2.41$	2.419	2.7.67
39	3.13	—	—	3.113	—	$7^2.11$	$3^2.71$	—	—	3.313
40	$2^3.5$	$2^2.5.7$	$2^4.3.5$	$2^2.5.17$	$2^3.5.11$	$2^2.3^3.5$	$2^7.5$	$2^2.5.37$	$2^3.3.5.7$	$2^2.5.47$
41	—	3.47	—	11.31	$3^2.7^2$	—	—	3.13.19	29^2	—
42	2.3.7	2.71	2.11^2	$2.3^2.19$	2.13.17	2.271	2.3.107	2.7.53	2.421	2.3.157
43	—	11.13	3^5	7^3	—	3.181	—	—	3.281	23.41
44	$2^2.11$	$2^4.3^2$	$2^2.61$	$2^3.43$	$2^2.3.37$	$2^5.17$	$2^2.7.23$	$2^3.3.31$	$2^2.211$	$2^4.59$
45	$3^2.5$	5.29	5.7^2	3.5.23	5.89	5.109	3.5.43	5.149	5.13^2	$3^3.5.7$
46	2.23	2.73	2.3.41	2.173	2.223	2.3.7.13	2.17.19	2.373	$2.3^2.47$	2.11.43
47	—	3.7^2	13.19	—	3.149	—	—	$3^2.83$	7.11^2	—
48	$2^4.3$	$2^2.37$	$2^5.31$	$2^2.3.29$	$2^6.7$	$2^2.137$	$2^3.3^4$	$2^2.11.17$	$2^4.53$	$2^2.3.79$
49	7^2	—	3.83	—	—	$3^2.61$	11.59	7.107	3.283	13.73

	0	1	2	3	4	5	6	7	8	9
50	2.5^2	$2.3.5^2$	2.5^3	$2.5^2.7$	$2.3^2.5^2$	$2.5^2.11$	$2.5^2.13$	$2.3.5^3$	$2.5^2.17$	$2.5^2.19$
51	3.17	—	—	$3^3.13$	11.41	19.29	3.7.31	—	23.37	3.317
52	$2^2.13$	$2^3.19$	$2^2.7.9$	$2^5.11$	$2^2.113$	$2^3.3.23$	$2^2.163$	$2^4.47$	$2^2.3.71$	$2^3.7.17$
53	—	$3^2.17$	11.23	—	3.151	7.79	—	3.251	—	—
54	2.3^3	2.7.11	2.127	2.3.59	2.227	2.277	2.3.109	2.13.29	2.7.61	$2.3^2.53$
55	5.11	5.31	3.5.17	5.71	5.7.13	3.5.37	5.131	5.151	$3^2.5.19$	5.191
56	$2^3.7$	$2^2.3.13$	2^8	$2^2.89$	$2^3.3.19$	$2^2.139$	$2^4.41$	$2^2.3^3.7$	$2^3.107$	$2^2.239$
57	3.19	—	—	3.7.17	—	—	$3^2.73$	—	—	3.11.29
58	2.29	2.79	2.3.43	2.179	2.229	$2.3^2.31$	2.7.47	2.379	2.3.11.13	2.479
59	—	3.53	7.37	—	$3^3.17$	13.43	—	3.11.23	—	7.137
60	$2^2.3.5$	$2^5.5$	$2^2.5.13$	$2^3.3^2.5$	$2^2.5.23$	$2^4.5.7$	$2^2.3.5.11$	$2^3.5.19$	$2^2.5.43$	$2^6.3.5$
61	—	7.23	$3^2.29$	19^2	—	3.11.17	—	—	3.7.41	31^2
62	2.31	2.3^4	2.131	2.181	2.3.7.11	2.281	2.331	2.3.127	2.431	2.13.37
63	$3^2.7$	—	—	3.11^2	—	—	3.13.17	7.109	—	$3^2.107$
64	2^6	$2^2.41$	$2^3.3.11$	$2^2.7.13$	$2^4.29$	$2^2.3.47$	$2^3.83$	$2^2.191$	$2^5.3^3$	$2^2.241$
65	5.13	3.5.11	5.53	5.73	3.5.31	5.113	5.7.19	$3^2.5.17$	5.173	5.193
66	2.3.11	2.83	2.7.19	2.3.61	2.233	2.283	$2.3^2.37$	2.383	2.433	2.3.7.23
67	—	—	3.89	—	—	$3^4.7$	23.29	13.59	3.17^2	—
68	$2^2.17$	$2^3.3.7$	$2^2.67$	$2^4.23$	$2^2.3^2.13$	$2^3.71$	$2^2.167$	$2^8.3$	$2^2.7.31$	$2^3.11^2$
69	3.23	13^2	—	$3^2.41$	7.67	—	3.223	—	11.79	3.17.19
70	2.5.7	2.5.17	$2.3^3.5$	2.5.37	2.5.47	2.3.5.19	2.5.67	2.5.7.11	2.3.5.29	2.5.97
71	—	$3^2.19$	—	7.53	3.157	—	11.61	3.257	13.67	—
72	$2^3.3^2$	$2^2.43$	$2^4.17$	$2^2.3.31$	$2^3.59$	$2^2.11.13$	$2^5.3.7$	$2^2.193$	$2^3.109$	$2^2.3^5$
73	—	—	3.7.13	—	11.43	3.191	—	—	$3^2.97$	7.139
74	2.37	2.3.29	2.137	2.11.17	2.3.79	2.7.41	2.337	$2.3^2.43$	2.19.23	2.487
75	3.5^2	$5^2.7$	$5^2.11$	3.5^3	$5^2.19$	$5^2.23$	$3^3.5^2$	$5^2.31$	$5^3.7$	$3.5^2.13$
76	$2^2.19$	$2^4.11$	$2^2.3.23$	$2^3.47$	$2^2.7.17$	$2^6.3^2$	$2^2.13^2$	$2^3.97$	$2^2.3.73$	$2^4.61$
77	7.11	3.59	—	13.29	$3^2.53$	—	—	3.7.37	—	—
78	2.3.13	2.89	2.139	$2.3^3.7$	2.239	2.17^2	2.3.113	2.389	2.439	2.3.163
79	—	—	$3^2.31$	—	—	3.193	7.97	19.41	3.293	11.89
80	$2^4.5$	$2^2.3^2.5$	$2^3.5.7$	$2^2.5.19$	$2^5.3.5$	$2^2.5.29$	$2^3.5.17$	$2^2.3.5.13$	$2^4.5.11$	$2^2.5.7^2$
81	3^4	—	—	3.127	13.37	7.83	3.227	11.71	—	$3^2.109$
82	2.41	2.7.13	2.3.47	2.191	2.241	2.3.97	2.11.31	2.17.23	$2.3^2.7^2$	2.491
83	—	3.61	—	—	3.7.23	11.53	—	$3^3.29$	—	—
84	$2^2.3.7$	$2^3.23$	$2^2.71$	$2^7.3$	$2^2.11^2$	$2^3.73$	$2^2.3^2.19$	$2^4.7^2$	$2^2.13.17$	$2^3.3.41$
85	5.17	5.37	3.5.19	5.7.11	5.97	$3^2.5.13$	5.137	5.157	3.5.59	5.197
86	2.43	2.3.31	2.11.13	2.193	2.3^5	2.293	2.7^3	2.3.131	2.443	2.17.29
87	3.29	11.17	7.41	$3^2.43$	—	—	3.229	—	—	3.7.47
88	$2^3.11$	$2^2.47$	$2^5.3^2$	$2^2.97$	$2^3.61$	$2^2.3.7^2$	$2^4.43$	$2^2.197$	$2^3.3.37$	$2^2.13.19$
89	—	$3^3.7$	17^2	—	3.163	19.31	13.53	3.263	7.127	23.43
90	$2.3^2.5$	2.5.19	2.5.29	2.3.5.13	$2.5.7^2$	2.5.59	2.3.5.23	2.5.79	2.5.89	$2.3^2.5.11$
91	7.13	—	3.97	17.23	—	3.197	—	7.113	$3^4.11$	—
92	$2^2.23$	$2^6.3$	$2^2.73$	$2^3.7^2$	$2^2.3.41$	$2^4.37$	$2^2.173$	$2^3.3^2.11$	$2^2.223$	$2^5.31$
93	3.31	—	—	3.131	17.29	—	$3^2.7.11$	13.61	19.47	3.331
94	2.47	2.97	$2.3.7^2$	2.197	2.13.19	$2.3^3.11$	2.347	2.397	2.3.149	2.7.71
95	5.19	3.5.13	5.59	5.79	$3^2.5.11$	5.7.17	5.139	3.5.53	5.179	5.199
96	$2^5.3$	$2^2.7^2$	$2^3.37$	$2^2.3^2.11$	$2^4.31$	$2^2.149$	$2^3.3.29$	$2^2.199$	$2^7.7$	$2^2.3.83$
97	—	—	$3^3.11$	—	7.71	3.199	17.41	—	3.13.23	—
98	2.7^2	$2.3^2.11$	2.149	2.199	2.3.83	2.13.23	2.349	2.3.7.19	2.449	2.499
99	$3^2.11$	—	13.23	3.7.19	—	—	3.233	17.47	29.31	$3^3.37$

	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
01	7	3	-	-	3	19	-	3	-	-	3	11	31	3	7^4	41	3	37	-	3
03	17	-	3	-	23	3	7	13	3	11	-	3	-	7	3	-	19	3	-	
05	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5
07	19	3	17	-	3	11	-	3	13	-	3	7	-	3	29	23	3	-	7	3
09	-	-	3	7	-	3	-	-	3	23	7	3	47^2	-	3	13	-	3	53^2	-
11	3	11	7	3	17	-	3	29	-	3	-	-	3	-	-	3	7	-	3	41
13	-	3	-	13	3	17	-	3	7	-	3	-	-	3	19	7	3	-	29	3
15	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5
17	3	-	-	3	13	37	3	17	23	3	-	29	3	7	-	3	-	11	3	-
19	-	3	23	-	3	7	-	3	17	19	3	13	7	3	41	11	3	-	-	3
21	-	19	3	-	7	3	-	-	3	17	43	3	-	11	3	-	-	3	7	23
23	3	-	-	3	-	-	3	-	-	3	7	11	3	23	-	3	43	7	3	37
25	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3
27	13	7	3	-	-	3	-	11	3	41	-	3	17	13	3	7	37	3	11	-
29	3	-	-	3	-	11	3	7	31	3	-	-	3	17	7	3	11	-	3	29
31	-	3	-	11^3	3	-	7	3	-	-	3	-	23	3	11	-	3	-	19	3
33	-	11	3	31	-	3	23	-	3	-	19	3	7	-	3	17	-	3	-	7
35	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5
37	17	3	-	7	3	29	-	3	11	13	3	-	-	3	-	43	3	7	-	3
39	-	17	3	13	-	3	11	37	3	7	-	3	-	-	3	-	7	3	17	-
41	3	7	17	3	11	23	3	-	7	3	13	-	3	-	-	3	19	-	3	17
43	7	3	11	17	3	-	31	3	19	29	3	-	-	3	7	-	3	13	-	3
45	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5
47	3	31	29	3	-	7	3	-	-	3	23	19	3	-	-	3	-	41	3	7
49	-	3	-	19	3	-	17	3	43^2	-	3	7	13	3	31	-	3	-	7	3
51	-	-	3	7	-	3	13	17	3	-	7	3	-	-	3	-	11	3	-	13
53	3	-	7	3	-	-	3	-	17	3	-	-	3	13	11	3	7	-	3	-
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57	7	13	3	23	31	3	-	7	3	19	11	3	37	-	3	-	-	3	-	-
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61	-	3	13	-	3	7	11	3	-	37	3	-	7	3	23	13	3	11	-	3
63	-	-	3	29	7	3	-	41	3	13	-	3	31	17	3	11	-	3	7	-
65	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5
67	11	3	7	-	3	-	-	3	-	7	3	11	-	3	-	17	3	-	47	3
69	-	7	3	37^2	13	3	-	29	3	11	-	3	-	23	3	7	17	3	19	-
71	3	-	31	3	-	-	3	7	-	3	19	13	3	-	7	3	-	17	3	-
73	29	3	19	-	3	11	7	3	-	-	3	41	-	3	-	31	3	47	13	3
75	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5
77	3	11	-	3	7	19	3	-	-	3	31	7	3	-	-	3	-		3	13
79	13	3	-	7	3	-	23	3	-	—	3	-	43	3	37	-	3	7	-	3
81	23	-	3	-	-	3	41^2	13	3	7	-	3	-	-	3	29	7	3	43	11
83	3	7	-	3	-	-	3	-	7	3	-	37	3	-	13	3	-	11	3	19
85	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3
87	-	-	3	19	-	3	7	-	3	-	-	3^7	-	7	3	13	-	3	-	29
89	3	29	-	3	-	7	3	-	-	3	-	11	3	-	19	3	-	-	3	7
91	-	3	-	13	3	37	19	3	31	11	3	7	29	3	47	-	3	-	7	3
93	-	-	3	7	-	3	-	11	3	-	7	3	-	-	3	-	-	3	11	41
95	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5
97	-	3	-	11	3	-	-	3	7	-	3	13^3	-	3	11	7	3	-	-	3
99	7	11	3	-	-	3	-	7	3	-	-	3	11	-	3	23	-	3	13	-

	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
01	-	7	11	-	19	3^2	13	-	7	47	-	3	-	11	3^2	7	43	3	-	13
03	7	29	-	3^2	41	31	3	7	-	3	-	11	3^2	13	7	19	-	-	3	-
05	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
07	31	13	3	-	-	7	-	11	3^2	-	-	3	7	59	13	-	17	3^2	11	7
09	17	-	-	3	7	11	3^2	-	13	3	19	7	23	31	-	3^2	11	17	7	-
11	-	17	13	7	3^2	-	23	3	37	-	7	-	-	3^2	11	13	29	7	17	3
13	23	11	7	-	-	3	-	47	31	7	-	3^2	11	19	3	-	7	3	-	17^3
15	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
17	7	3	-	31	17	-	-	7	11	-	13	23	-	3	7	-	3^2	53	-	11
19	-	-	29	-	13	3^2	7	-	19	-	-	3	-	7	3^2	-	31	11	61	-
21	19	-	-	3^2	11	7	17	61^2	-	3	-	13	7	29	-	11	-	-	3	7
23	-	3^2	11	-	7	13	-	17	-	-	3^2	7	41	11	-	-	23	-	7	3^2
25	5	5^5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
27	3	53	7	3	23	-	13	-	43	7	-	-	3	-	19	3^2	7	29	3	13
29	13	7	-	-	3^2	-	19	11	7	-	17	-	-	3^2	43	7	3	-	11	31
31	7	31	3^2	-	47	11	-	7	3	-	29	17	-	61	7	23	11	19	-	-
33	3^2	13	53	11	-	-	7	-	-	19	37	-	17	7	11	3	41	-	3^2	-
35	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
37	-	-	13	47	7	3^2	-	37	3	31	11	7	19	-	17	13	-	3	7	-
39	3	43	41	7	19	-	3	-	11	13	7	-	3^2	-	23	17	-	7	3	11
41	-	3^2	7	13	31	-	11	29	23	7	3^2	41	-	3	-	19	7	11	47	3^2
43	17	7	23	-	11	3	-	19	7	-	13	3	-	43	3	7	-	3^2	29	-
45	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
47	11	3	17	-	3^2	-	7	3	-	-	19	11	31	7	-	-	3	47	37	17
49	-	47	19	17	-	7	41	23	3	11	-	3^2	7	-	3	-	-	3	13	7
51	3^2	23	-	3	7	53	3	11	-	3^2	-	7	13	19	-	37	-	-	7	-
53	43	3	-	7	3	11	13	3^2	-	59	7	-	-	3	61	29	11	7	23	13
55	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
57	3	7	-	3^2	-	-	23	13	7	3	-	-	11	-	-	7	-	67	3	-
59	7	13	-	-	3	-	-	7	17	37	11	-	-	3	7	47	3	-	43	3^2
61	-	29	3	-	-	3	7	-	11	17	31	19	-	7	3	-	59	3^2	-	11
63	3	-	13	19	-	7	11	53	-	3	17	23	7	-	-	3^2	-	11	3	7
65	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
67	-	-	11	7	-	29	19	-	3	-	7	3^2	17	11	3	-	13	7	31	-
69	11	-	7	3	-	43	3	-	53	7	13	11	3	17	41	3	7	19	3^2	-
71	37	7	-	-	13	-	-	3^2	7	11	23	43	-	31	17	7	3^2	13	-	3
73	7	19	3	-	23	3^2	-	7	3	29	-	13	-	-	7	17	-	37	11	-
75	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
77	17	3^2	29	11	19	7	-	3	-	41	3^3	-	7	3	11	23	3	17	-	7
79	-	11	3	31	7	3	13	-	3^2	23	-	7	11	29	3	19	-	3^2	7	13
81	13	-	17	7	59^2	-	3^2	19	-	3	7	37	3	13	-	3^2	31	7	3	17
83	-	3	7	17	3^2	-	29	13	11	7	3	47	-	3^2	-	-	7	-	19	11
85	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
87	7	-	19	3	11	17	3	7	13	3^2	61	53	3	41	7	11	43	-	3^2	-
89	-	3	11	-	3	37	7	3^2	-	-	29	59	-	7	67^2	13	3^2	-	-	3
91	11	-	3	-	-	7	-	17	3	13	-	11	7	-	3^2	-	-	3	67	7
93	3	31	37	13	7	-	3	-	17	11	-	7	3^2	23	-	3	13	-	7	-
95	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
97	19	23	7	43	13	11	-	-	3^2	7	17	3	-	-	3	-	7	3^2	59	19
99	3	7	-	11	-	59	3^2	29	7	31	-	13	3	53	11	7	37	-	23	-

	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69
01	3	-	7	3^2	11	-	3	-	-	7	17	-	3^2	-	37	11	7	-	3	67
03	-	3^2	11	-	3	-	13	3	7	-	3^2	17	-	11	19	7	31	-	-	3^2
05	7	5	3.5	5	23	3.5	19	7	3^2	5	5	3.5	17	13	3.5	5	5	3^2	5	5
07	3	-	41	29	-	-	3^2	13	-	11	-	31	3	7	43	3^2	-	19	3	-
09	-	13	-	-	3^2	7	71	11	37	19	3	41	7	3^2	13	23	3	-	11	7
11	-	19	3^2	47	7	11	31	-	13	23	-	3^2	-	-	3	17	11	3	7	-
13	3^2	-	13	7	-	37	3	29	-	3^2	7	-	19	59	11	13	17	7	3^2	31
15	17	11	7	5	3.5	5	5	3^2	5	7	3.5	5	11	3.5	5	5	3^2	17	29	3.5
17	29	7	37	13	-	3^2	41	-	7	61	11	3	-	-	3^2	7	13	3	17	-
19	7	-	17	3^2	-	-	3	7	11	3	13	29	3^2	71	7	41	-	-	3	11
21	-	3^2	23	17	13	-	7	3	-	31	3^2	-	-	7	-	-	3	11	19	3^2
23	-	47	3	-	11	7	-	59	3^2	-	19	13	7	-	3	11	37	3^2	-	7
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33	7	29	-	-	3	11	43	3^2	19	17	3	-	23	3	7	47	3^2	-	-	3
35	19	13	3.5	11	5	3^2	7	31	3.5	5	17	3.5	29	7	3^2	5	5	3.5	5	19
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41	71^2	53	3	7	-	3	-	-	3^2	13	7	23	79^2	17	19	31	29	3^2	-	11
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47	7	-	3^2	-	13	3	-	7	3	19	-	3^2	-	11	7	-	17	13	41	-
49	3^2	19	29	3	-	31	7	-	-	3^2	23	11	3	7	-	37	61	17	3^2	-
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71	11	-	7	41	-	3^2	53	29	19	7	13	11	-	23	3^2	-	7	37	-	-
73	19	7	-	3^2	13	-	31	23	7	11	-	-	3^2	-	-	7	-	13	29	19
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77	-	31	3	19	-	11	7	53	3^2	43	59	29	-	7	17	-	11	3^2	23	-
79	3	-	-	11	-	7	3^2	-	-	3	-	37	7	-	11	3^2	-	-	3	7
81	-	11	-	-	3^2	-	13	41	-	-	3	7	11	3^2	-	-	17	-	7	13
83	13	71	3^2	7	-	3	-	-	37	31	7	3^2	61	13	3	29	41	7	-	-
85	3^2	17	7	3.5	5	5	3.5	13	11	3^2	5	5	3.5	5	5	3.5	7	23	3^2	11
87	-	7	17	-	31	37	11	3^2	7	-	3	23	-	3	13	7	3^2	11	71	17
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95	5	5	3.5	13	7	3.5	17	19	3^2	11	23	3.5	5	5	3.5	5	13	3^2	7	5
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99	-	3	7	-	3^2	11	41	3	17	7	19	-	-	3^2	67	-	7	13	-	3

	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89
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07	13	23	-	-	3^2	-	-	3.7	37	-	17	67	29	3^2	7	47	19	-	-	3
09	43	-	3^2	-	31	3	7	13	19	11	-	3^2	-	7	3	67	-	3	23	59
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15	23	5	3.5	5.7	5	3^2	5	5	3.5	5	5.7	3.5	31	5	3^2	13	5	3.5	41	5
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19	-	3^2	-	13	3	73	19	31	7	-	3^2	23	-	47	-	7	13	-	-	3^2
21	17	-	29	-	41	23	-	7	3^2	89^2	13	3	-	53	3.7	-	37	3^2	-	11
23	3	17	31	3	13	-	3^2	-	-	19	71	-	3	29	-	3^2	-	13	17	-
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37	31	3^2	-	23	37	-	7	3	17	-	3^2	79	-	3.7	13	-	3	-	-	3^2
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79	-	3	29	47	3^2	13	7	3	-	79	3	-	17	3^2	61	23	11	-	13	41
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93	41	-	11	-	59	3	7	-	3^2	-	-	3	-	7	19	13	-	3^2	-	17
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97	47	3	-	13	3^2	71	43	23	53	11	3	7	-	3^2	29	-	13	19	31	3
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	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109
01	-	19	3	71	17	3	-	89	11	-	73	3.7	101^2	-	3	-	-	29	7	11
03	3	-	-	3.7	-	13	11	31	-	3	7	-	19	-	101	3^2	23	11	13	-
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07	-	7	11	41	23	3	13	17	3.7	-	-	3^2	59	11	3	19	-	43	101	13
09	11	-	-	29	97^2	37	3	19	17	3^3	-	11	41	13	7	31	103^2	-	3^2	-
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25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
27	17	-	-	3	11	7	3	71	31	3^2	37	13	3.7	23	-	11	-	17	3^3	7
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33	3	-	7	17	-	-	13	-	-	3.7	79	-	3^3	-	-	3	31	-	23	13
35	13	3.5	5	5	3.5	5	41	3.5	5.7	5	3.5	5	23	3.5	5	5.7	3.5	19	11	3.5
37	7	-	3	-	-	11	23	13	3^2	19	-	31	29	-	3.7	41	11	3^2	-	-
39	23	19	-	11	-	-	3.7	-	-	3	-	-	3	7	13	3^2	-	-	3	-
41	-	11	-	-	3^2	29	31	17	13	-	3	-	19	3^3	53	83	3	23	37	3.7
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49	-	7	3	-	11	3^2	-	-	3.7	-	13	17	37	79	3^5	11	23	3	19	-
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57	3	-	-	3	7	19	29	11	-	3	89	7	13	-	-	17	-	31	3.7	-
59	-	43	47	7	3^2	79	13	3	-	23	3.7	-	-	3^2	-	-	11	29	-	13
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63	19	17	59	3	-	73	3	13	7	3^4	29	-	11	43	-	3.7	-	47	17	19
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67	-	89	3	17	-	3^2	7	-	11	-	-	3	-	7	3^2	-	-	37	-	11
69	3	53	13	3^3	17	7	11	-	71	3	-	-	3.7	-	19	13	47	89	3	7
71	47	3^2	73	-	11	17	19	3	-	13	3^3	7	-	3	37	31	3	-	7	23
73	43	-	11	13	-	3	17	29	3^2	-	7	3	-	23	3	97	13	3.7	83	-
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81	3^2	-	-	53	19	13	3.7	-	41	3^2	17	-	23	7	47	3	11	-	13	79
83	31	3	-	11	29	37	23	3^2	-	67	3	17	13	3	11	19	3^2	41	-	3.7
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	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126
01	19	17	23	3	13	31	3^2	-	-	3	11	-	3.7	-	-	3^3	-
03	-	3	17	89	3.7	-	41	47	29	-	3	13	-	3^2	79	-	3
05	31	5	3.5	5.7	5	3.5	5.11	5	3.5	5	5.7	3.5	5	23	3.5	41	5
07	3^2	29	7	3	17	37	53	23	-	3.7	-	-	13	31	19	3.11	7
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19	3	-	13	3.7	19	-	3^2	-	53	29	17	-	3	97	11	13	-
21	103	337	7	-	3^3	41	-	3	-	13	3	17	101	3^2	-	19	3.7
23	73	7	29	13	-	23	59	19	3.7	-	11	3^3	17	-	41	7	13
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
27	-	3	103	47	13	-	7.11	3^2	-	-	19	67	-	3.7	17	-	23
29	41	31	19	-	11	3.7	29	37	3	79	23	13	7	-	3^2	17	73
31	3	-	11	3^2	23	13	3	-	-	41	53	7	3^3	19	31	3	17
33	17	3^2	47	7	37	19	-	3	-	-	3.7	11	13	3	-	83	3
35	5	17	3.5	5	5	3.5	13	5	3.5	5.7	29	3.5	5	5	3.5	23	5.7
37	13	37	17	3	-	83	3^3	97	19	23	-	53	3	13	-	3.7	-
39	19	47	-	23	31	11	103	3.7	-	-	3	61	-	3^3	7	-	3.11
41	61	13	3^2	11	17	3	7	59	3	-	-	19	-	41	3.11	-	-
43	3^3	11	-	19	-	97	3	-	13	3^2	-	-	3.7	-	23	37	47
45	5	3.5	13	5	3.5	5	17	3.5	23	5	3.5	5.7	31	3.5	19	13	3.5
47	-	71	23	7	-	3^2	19	17	359	13	7	3	37	-	3^3	-	-
49	29	-	7	13	107^2	-	353	31	41	3.7	-	-	3^2	53	59	47	13
51	43	3.7	-	-	3.11	-	61	3	7	19	13	29	-	23	-	7	3
53	7	19	31	-	13	3	43	23	3^3	-	17	3	-	11	3.7	-	-
55	3.5	23	5	3.5	29	5	3.5	5	5	3.5	5	5.11	3.5	5.7	47	3.5	5
57	-	3	-	41	19	13	-	3	71	11	3	-	103	3^2	-	29	3
59	-	-	3^3	37	7	3	89	11	59	-	31	3.7	13	17	3	19	-
61	3^2	-	-	3.7	73	11	13	19	29	3^3	7	-	61	47	17	53	11
63	13	3	7	11	3	31	107	3^2	-	7	3	-	-	13	103	17	3.7
65	5	5.7	3.5	5	5	3.5	5	13	3.5	5	19	3.5	5.11	5	3.5	5.7	17
67	3.7	13	19	3^3	-	43	3	41	-	3	11	23^3	29	83	13	59	53
69	-	17	59	-	3	23	7	3	13	-	3^3	43	-	3.7	37	-	41
71	-	-	13	83	-	3.7	11	79	3^2	-	-	3	7	89	3	13	-
73	3	-	-	17	7.11	71	3^2	61	31	13	-	37	3	-	-	3.11	19
75	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	19	-	3.7	31	23	17	-	-	37	29	13	3.11	-	-	3	-	7
79	3^2	7	-	3	13	-	17	-	7	3.11	47	19	3	-	-	3.7	31
81	7	3	29	19	43	37	-	3.7	109^2	-	3	13	-	3	7	23	3^2
83	-	53	3	-	-	13	7	-	17	23	43	31	71	29	19	-	11
85	3.5	5	37	3.5	5	5.7	3.5	5	5	17	5	5	3.5	5	5.11	3.5	43
87	-	113	-	59	3.7	-	13	3	-	-	17	7	11	3	-	41	3
89	13	67	53	7	-	3	-	-	3^2	19	7.11	17	-	13	23	-	-
91	3	19	7	3	-	67	3^3	13	23	3.7	107	73	17	-	-	3^2	37
93	-	3.7	23	-	3^2	-	11	3	7	67	29	89	19	17	13	7	3
95	5.7	5	3.5	43	5.11	3.5	5	5.7	3.5	5	41	3.5	5	37	17	5.11	5
97	3^3	-	13	29	-	-	3.7	47	-	31	-	-	3	23	-	13	-
99	11	3	-	-	3	7	-	19	73	13	37	11	7	3	29	43	17

	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143
01	13	17	19	-	3.11	43	47	3^2	23	29	3	37	-	3.13	59	11	3.7
03	-	31	3.11	-	-	3^4	53	13	3.7	61	71	43	-	19	3^2	7	-
05	3.5	13	29	3.5	5	19	3.5	5.7	37	3.5	5	5.11	3.5	5	5.7	3.5	5
07	97	3^2	-	-	17	47	7	41	13	11	3^2	-	-	3.7	-	-	19
09	71	-	13	-	-	3.7	-	23	19	31	-	3	7	-	3	13	41
11	19	23	-	3	7	11	17	-	59	3.13	-	7	3	-	103	3^2	11
13	-	3	37	7.11	31	73	-	17	-	-	3.7	19	-	3^2	11	61	3.13
15	5	5.11	3.5	19	43	3.5	5	5	3.5	5.7	5.13	3.5	5.11	5	3.5	5	5.7
17	3^4	7	-	3	13	-	23	-	7	17	29	41	3	107	19	3.7	103
19	23	3	-	47	3	-	19	3.7	11	-	17	13	31	3	7	59	37
21	-	-	59	29	-	3.13	7.11	-	3	53	-	17	-	7	3^3	-	-
23	3	-	-	3^2	11	7	3	31	-	19	-	23	3.7	37	29	3.11	-
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
27	13	101	31	7	-	3	-	29	3^4	-	37	3.11	19	83	17	41	-
29	3	-	7	43	19	-	3^2	13	83	3.7	-	-	3	-	71	3^3	23
31	29	3.7	67	83	3^2	101	-	3.11	7	43	23	-	-	3^2	13	19	17
33	17	41	3^3	-	23	3.11	67	19	3.13	-	31	29	-	-	3.7	43	11
35	3.5	17	13	3.5	37	5	3.5	5	5	3.5	41	5	3.5	5.7	5.11	3.5	47
37	47	3.11	17	-	29	31	-	3^2	-	13	19	101	7.11	3	67	23	3^5
39	-	37	19	17	7	3^2	-	89	3	23	11	3.7	53	101	3^2	29	13
41	31	-	-	3.7	17	-	3	-	11	3	7.13	-	3^2	19	79	47	-
43	-	3^2	43	-	3.13	17	11	3	29	7	3^3	109	73	31	-	-	3.7
45	5	5.7	3.5	5	5.11	3.5	17	5	3.5	5	5	3.5	5	5	3.5	5.7	19
47	3.7	29	107	3	-	13	3^2	17	19	3	59	61	3	11	43	3^2	-
49	61	3	23	-	3^3	-	7	3	17	-	3	11	29	3.7	-	-	3
51	41	71	3^2	31	-	3.7	79	-	3	17	-	3^6	7	-	53	-	113
53	13	-	-	19	7	29	3	11	-	37	17	7	3	23	-	3	31
55	5	3.5	5	5.7	3.5	5.11	5	3.5	5	5	3.5	17	5	3.5	19	5	3.5
57	-	13	3.7	11	59	3^3	19	-	3	7	-	31	17	-	3.11	53	7
59	3	7	-	3^2	-	-	61	43	7.13	29	-	-	3.11	17	-	3.7	83
61	7	3^2	13	37	41	89	31	3.7	71	19	3.11	83	23	43	17	13	3
63	-	19	29	-	-	3	23	-	3.11	13	-	3	-	41	3	17	53
65	3.5	31	5	3.5	5	5.7	3.5	5	5	3.5	5	47	3.5	29	5	3.5	5.13
67	17	3	-	73	3.7	-	-	3	-	79	3.13	7	-	3^3	31	11	3
69	113^2	17	3.11	7	13	3	29	-	3	-	7	23	61	11	3	19	-
71	3.11	61	109	3	-	23	3	19	41	3.7	47	97	3	-	37	67	7
73	53	3.7	-	17	3	13	43	3^3	7	113	3	-	89	3	-	7	3^2
75	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	3	79	19	3^2	-	17	3.7	-	-	47	23	-	3^2	7	-	3	11
79	13	3^5	-	11	23	7	17	3	37	-	3^2	-	7	19	11	109	3
81	-	11	3	103	7	19	-	17	3^3	-	-	3.7	31	-	29	-	73
83	3	13	-	3.7	-	37	3^2	97	17	3	7.11	-	59	-	13	3^3	19
85	5	3.5	5.7	5	3.5	5	5	3.5	5.11	5.7	3.5	5	5	3.5	5	5	3.5
87	19	7	3^3	23	-	43	11	-	3.7	-	17	3^2	71	-	3	7.13	-
89	3.7	-	31	3	109	97	3	41	107	3^4	-	17	3	73	7	3.11	-
91	-	3	11	19	3	-	7	3^2	-	-	3	29	17	3.7	23	31	3^3
93	11	-	61	-	79	3.7	59	103	23	-	13	3.11	7	17	19	-	37
95	3.5	5	23	3.5	5.7	5	3.5	5	5	3.5	31	5.7	3.5	5	17	3.5	5
97	67	3^2	41	7	53	-	-	3.11	-	-	3.7	13	-	37	-	17	3
99	-	-	3.7	-	67	3.11	-	-	3^2	19	-	41	-	23	3	79	17

	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
01	–	17	31	61	19	3	7	–	3^3	107	–	3	–	7	23	–	–
03	3	–	17	3.13	113	7	3^2	11	23	3	73	37	3.7	41	–	3^3	13
05	43	3.5	23	17	3.5	5.11	5	19	5	5	3.13	5.7	5	3^2.5	29	5	5.11
07	–	89	3^3	7.11	17	3	43	–	37	–	31	3^2	–	113	3.11	–	–
09	3^2	11	7	3	59	17	3	29	67	3^7	19	13	3.11	23	–	3	7
11	–	3.7	19	47	3	31	17	23	41	61	3.11	–	67	3	97	7	3^3
13	29	23	3	–	–	3^2	–	17	3.11	–	–	3	13	19	3.7	–	67
15	3.5	5	37	3.5	5	19	5.7	5	17	3.5	5	29	3^2.5	5.7	5	3.5	5
17	13	3^2	47	–	3.11	7	–	3	–	17	3^3	59	23	3.13	–	11	19
19	–	–	3.11	41	73	3	23	13	19	–	17	3.7	–	11	3	–	83
21	3.11	13	–	3.7	–	43	3^2	–	31	3	7	17	41	79	13	29	37
23	–	47	7	–	3^5	–	83	3	13	7.11	53	19	17	3^2	–	–	3.7
25	5^2	5^2	5^3	5^2	5^2	5^2	5^2	5^3	5^2	5^2	5^2	5^2	5^6	5^2	5^2	5^2	5^2
27	3.7	73	–	3	–	23	3	7	–	3.13	–	–	3	–	17	3	31
29	47	29	–	103	3	–	19	3^2	97	–	37	53	–	3.7	11	17	3.13
31	–	11	3	–	–	3.7	–	–	3	–	13	31	29	–	3^2	89	17
33	17	–	–	3^2	7.13	109	3	37	–	19	23	7	3^4	–	71	47	–
35	5	3.5	5	5.7	3.5	29	31	3.5	5.11	5	5.7	5.13	53	3.5	5	5	3.5
37	–	–	3.7	–	37	3.13	11	–	3^2	7	43	3	19	–	3	–	29
39	3	31	–	17	19	–	3^3	–	7	3	–	41	3.13	–	47	3.7	43
41	7	37	11^4	–	17	67	89	3.7	–	23	3	–	–	3^3	31	19	3
43	101	–	3^2	23	–	17	7	19	3	67	–	3.11	–	7.13	3	107	61
45	3.5	5	29	3.5	5	5.7	17	5.13	5	5.11	5	5	5.7	47	5	3.5	5
47	–	3.13	97	–	3.7	–	41	3^4	79	103	19	7	–	29	23	37	3^2
49	–	–	19	43	31	3.11	101	–	3.13	–	7	71	–	–	3^3	41	11
51	3	–	23	3.11	–	–	29	109	101	3.7	–	–	37	19	11	3.13	7
53	97	3.7	–	–	3	19	–	3	7	13	17	103	11	59	83	43	3
55	5.7	41	3.5	5.13	5	3.5	5	5.7	3^2.5	37	5.11	17	31	23	5.7	5	5.13
57	61	–	–	3	83	–	3.7	23	19	3	29	47	17	7	101	3^4	–
59	19	23	107	–	3.13	7	37	31	–	–	3	–	7	17	–	–	53
61	–	–	3^4	29	7	3	–	–	3	–	–	3.7	–	–	17	11	–
63	3^2	–	31	3.7	89	13	3	59	–	3^3	47	79	23	11	29	17	–
65	5.11	3.5	5.7	5	3.5	41	23	3^2.5	43	5.7	3.5	5.11	5.13	3.5	19	31	5.7
67	17	7	3	–	–	3^2	19	29	3.7	11	–	3	–	–	41	7	–
69	3.7	17	–	3^3	–	–	3	7.11	–	47	31	–	3^2	13	7	3	–
71	29	3^2	17	–	3	11	7	3.13	–	19	3^4	23	–	3.7	59	–	3.11
73	41	19	67	17	107	3.7	–	–	3^2	–	–	29	7	–	3.11	–	–
75	5^2	5^2	5^2	5^2	5^3	5^2	5^2	5^2	5^2	5^3	5^2	5^2	5^2	5^2	5^3	5^2	5^2
77	31	43	13	7	3^3	17	–	3	–	–	3.7	37	61	3^2	–	13	23
79	–	61	3.7	–	–	3	17	43	3.11	7.13	23	3^3	–	31	67	19	7
81	3^2	7	53	3.13	23	71	3.11	17	37	3^2	113	–	3	43	–	3.7	13
83	7	3	–	–	3.11	–	–	3.7	17	–	3.13	–	–	3	7	11	3^2
85	5	5	3.5	5	5.13	3.5	5.7	5	3.5	17	19	3.5	5	5.7	3^2.5	23	5
87	3.11	29	19	31	–	7	47	–	–	23	17	109	3^3	–	–	3	–
89	–	3^2	37	23	3.7	13	79	61	–	11	3^2	17	29	19	–	59	31
91	43	–	59	7	–	19	–	11	3^2	–	7	3	17	–	3	–	–
93	3	–	7	3	53	29	3^3	–	41	3.7	–	31	3	17	23	3^2	19
95	5.13	3.5	5	5.11	3.5	5	5	3.5	5.7	5	3.5	5	43	5.13	5.11	5.7	29
97	19	11	23	–	–	3	31	7.13	3	89	–	3^2	11	–	3.7	17	–
99	3^4	13	–	3	47	53	3.7	–	–	29	11	19	3	37	13	3	17

	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177
01	3^2	17	-	3.7	29	13	19	53	-	3^2	7	103	73	-	37	3	31
03	-	3.11	17	47	3	-	-	3^2	-	7	3	-	11.13	3	23	29	3.7
05	5	5.7	3.5	17	5	$3^2.5$	5.13	5	5.7	19	5.11	31	5	5	$3^2.5$	5.7	5
07	3.7	19	23	3^2	17	-	3	7^5	29	3	-	-	3^2	103	41	3	-
09	89	3^2	47	61	3	17	31	3.13	37	73	3^2	-	19	3.7	-	-	3
11	-	29	3	-	19	3.7	17	-	3^2	-	71	3	7	23	3.13	11	89
13	41	31	11	3	7	37	3^3	17	13	53	109	7	29	11	83	19	-
15	5.11	23	5.13	5.7	$3^2.5$	5	5	19	17	41	5.7	5.11	5	$3^2.5$	31	5.13	3.5
17	71	-	3.7	-	83	29	73	67	3	7.11	-	3^2	-	-	3	79	7
19	3^3	7	-	3.13	-	-	3	11	7	31	19	67	23	-	-	3.7	47
21	47	3	19	-	3	11	23	3^3	-	-	3.13	17	-	3	7	67	3.11
23	23	-	3	11	31	3^2	7	-	3	29	-	3	17	19	3^3	-	37
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
27	-	3^3	29	-	3.7	13	43	71	-	-	3.11	23	-	37	17	-	19
29	127^2	-	3	7	-	23	-	-	3^3	-	7	3	31	29	3	17	-
31	19	-	7	3	61	-	3.11	-	-	3.7	37	-	53	-	47	3^2	7.17
33	17	3.7	-	-	3.11	-	29	31	41	-	3	19	-	3.13	89	7.11	23
35	5.7	17	5.11	19	5	3.5	5	5.7	3.5	5	23	$3^2.5$	5	5.11	5.7	5	5
37	3.11	13	17	3	23	127	3.7	113	-	3^3	-	11	3	47	19	3	-
39	-	3	-	17	37	7	19	3^2	13	11	29	-	7	3	-	31	3^3
41	-	109	3.13	41	17	3^2	-	11	3	-	61	3.7	-	107	3^2	23	113
43	3	37	59	3^3	71	17	3	-	-	3.13	31	43	41	-	53	3	11
45	5	19	5.7	5.11	3.5	5	17	3.5	5	5.7	$3^2.5$	5	5	3.5	5.11	5	5.7
47	67	7.11	3	-	-	31	-	17	3.7	-	13	3	19	73	3	7	-
49	3.7	-	-	3	19	-	3^2	29	17	3	11	47	3	-	23	37	-
51	31	3	83	-	3^3	-	7	41	23	59	3	13	-	3.7	-	19	61
53	29	-	23	-	-	3.7	11	19	3	-	17	3^3	37	31	3	127	41
55	$3^2.5$	5	5	3.5	5.7	5	3.5	5	5	$3^2.5$	47	5.7	5.13	5	5	5.11	53
57	107	3	11	7	3	-	13	3^2	31	37	3.7	-	17	3.11	97	-	3^2
59	113	71	3.7	109	29	3^3	-	23	3	7	-	3.11	-	79	3^2	-	43
61	3	23	-	31	-	-	37	13	7	3.11	131^2	41	3^3	19	17	3.7	-
63	7	3.13	-	101	3	19	-	3.7	-	113	3^2	61	97	3	7.13	17	31
65	53	5	3.5	37	5	5.11	5.7	5	$3^2.5$	5	5	3.5	23	5.7	3.5	5	5.11
67	17	-	13	3.11	-	7	3^3	101	19	3	-	31	3.7	-	11	3.13	109
69	19	17	-	43	3.7	79	41	3	71	101	59	7	11	3^3	-	-	3
71	103	53	17	7.13	73	3	31	-	3	43	7.11	19	29	-	3	41	13
73	3^2	-	7	17	-	-	3	47	11	3.7	13	23	3	101	-	43	7
75	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	7	41	53	-	11	17	19	7	3	-	89	3.13	-	-	31	11	29
79	3	73	11	3^2	59	13	3.7	-	-	3	41	37	3^2	7.11	-	71	23
81	11	3^3	-	-	3	7	97	3.17	-	19	23	11	7.13	3	-	-	3
83	-	19	43	53	23	67	13	-	3^3	11	-	3.7	-	-	3	-	-
85	5.13	5	29	5.7	31	47	3^2 5	5.11	43	5.17	5.7	5	19	5.13	5	$3^2.5$	5
87	-	61	7	-	19	37	-	3.13	-	7	3.17	59	-	29	43	23	3.7
89	-	7.13	3^3	11	53	3	103	-	3.7	23	-	3.17	-	-	3.11	19	-
91	3.7	11	37	23	47	-	29	19	13	3^3	-	-	3.17	-	7	3	-
93	-	3	97	-	3	-	7	3^2	-	-	3.11	-	-	3.7	73	13	3^3
95	41	5	3.5	5	5	5.7	5	31	5.11	5.13	19	3.5	5.7	5	$3^2.5$	5	5
97	3	43	19	3^3	7	59	3.11	61	23	41	29	7	3^2	-	-	3.17	37
99	97	3^3	23	7	3.11	-	107	43	89	-	3^3	-	127	19	-	11	3.17

	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193
01	7	3^3	47	23	3	–	–	3.7	19	–	3^2	41	–	3	7.13	–
03	19	–	3.17	43	109	3	7.11	3^2.5	–	3^3	59	–	3	31	7	37	97
05	3.5	5	13	3.17	5.11	5.7	3^2.5	5	5	29	5	19	5.7	5	23	5.11
07	–	47	11	19	3.7	–	79	31	23	13	3	37	83	3.11	–	43
09	11	–	3^3	7.13	131	3.17	41	83	3	53	7	3.11	–	97	3.19	–
11	3^2	–	31	3	–	–	3.17	107	37	3^3	13	–	3	29	–	41
13	47	3.7	–	59	3.13	–	–	3.11	7	–	3	–	–	23	–	31
15	5.7	5	3.5	5	5	5.11	29	5.7	5.17	19	53	5.13	5	5	5.7	5
17	3	19	43	3^3	–	13	3.7	–	–	3.17	31	–	3^2	7	11	47
19	103	3.11	37	–	3	7	113	3	43	–	3^3	–	7.13	3	–	–
21	71	–	3	–	19	31	109	–	3^2	97	59	3.7	23	–	43	139^2
23	3.13	–	67	3.7	–	73	23	–	11	3	7	127	3.17	13	47	3.19
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5	5^2	5^2	5^2
27	–	7.13	3^2	–	11	41	–	97	3.7	61	67	3^3	53	31	3.17	7.11
29	3.7	–	11	3	–	–	3	7	13	3^2	19	23	3	37	41	3.17
31	11	43	19	–	59	23	7	29	31	–	3	11	–	3.7	–	13
33	17	79	3	–	–	3^3	–	43	3	131	37	3	7	53	3^2	–
35	29	5.17	5	5.13	5.7	19	3.5	5.11	5	3.5	5	5.7	3^2.5	43	5	3.5
37	–	3^2	17	7	3	11	103	37	–	41	3.7	29	–	3	–	61
39	–	–	3.7	97	23	3	–	–	19	7	–	59	79	–	3.11	83
41	19	7.11	–	3	29	–	3^3	–	7	3	83	31	3.11	–	71	3.7
43	7	3	–	–	3^2	83	–	3.7	103	–	3.11	19	137	3^3	7	23
45	43	37	3^2.5	19	41	3.5	5.7	5	5.11	23	5	3^2.5	5.13	5.7	3.5	53
47	3^3	131	–	23	71	7	3.11	17	29	3^2	47	–	3.7	41	19	3
49	13	31	–	–	3.7	59	19	3^3	17	–	61	7	43	3.13	–	11
51	–	29	3.11	7	–	3^2	–	13	3	17	7	3	–	11	3^3	37
53	3.11	13	7	3^2	–	–	3	–	23	3.7	17	11	29	107	13	3
55	5	5.7	23	5	3.5	5	5	3.5	5.7	5.11	3^2.5	5.17	37	3.5	5	5.7
57	7	–	3.13	67	–	29	–	7.11	3^3	–	109	71	59	–	3.7	13
59	3	–	–	3	19	11	3.7	67	47	3.13	–	–	3	23	–	3^3
61	53	3	–	127	3^2	43	–	23	–	73	3	67	7	3^2	103	19
63	–	23	3^3	41	7	3	37	19	3	29	13	3.7	11	–	3	67
65	3^3.5	5	5	5.7	5.13	5	3.5	47	5	3^2.5	5.7	5	31	5	5	3.5
67	17	53	89	37	3	–	59	3^2	11	7	19	13	23	3	–	107
69	107	7.17	19	–	–	3.13	23	31	3.7	137^2	–	3	–	29	3^2	7
71	3.7	–	17	3^3	11	–	47	7	–	3	113	61	13	19	7	3.11
73	61	3^2	31	17	3	19	29	41	71	–	3^3	–	–	3.7	–	–
75	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	59	–	–	73	7	23	3^2	13	19	3.11	43	7	3	127	37	3^2
79	19	3.13	101	53	3^3	–	17	3.11	–	89	3.7	–	–	3^2	13	–
81	–	–	3.7	–	101	3.11	–	17	3.13	7	79	3^3	–	–	3	–
83	3^2	7	107	3.11	47	31	61	–	7.17	3^2	23	41	3	–	11	3.7
85	5.7	5.11	5	5	23	5	5	5.7	37	5.13	3.5	5	5.11	3.5	5.7	5
87	31	–	3	13	–	3^3	19	–	3	–	101	3	–	7	3^2	–
89	67	–	–	43	–	37	3	29	11	3	13	17	3^3	31	–	23
91	–	3^2	79	–	3.7	53	41	3	–	19	3^2	7	17	3	101	–
93	29	19	37	23	11	3	–	–	31	–	7	3.13	61	17	59	41
95	3.5	59	5.7	3.5	5	5.13	3^2.5	5	5	5.7	5	29	5.19	5.11	5.17	3^2.5
97	11	3.7	–	31	19	–	53	3	7	–	3	11	113	3^3	23	7.17
99	7	41	3^2	–	29	3	13	7	23	11	–	3^2	71	73	3.7	19

	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209
01	29	-	17	3.11	-	7	59	-	-	67	23	83	3^3	127	31	3
03	-	3.11	-	61	3.7	13	83	3	89	79	3^2	29	11	67	71	-
05	5	47	3.5	5.7	5.17	3.5	5	5	3^2.5	31	5.7	3.5	5.13	41	5.19	37
07	3	-	7	3	29	17	3^3	-	11	3.7	-	-	3	-	-	23
09	13	3.7	-	-	31	43	107	3	7	23	3	-	37	3^3	-	29
11	47	109	3^2	23	11	3	-	7.13	3	19	-	43	-	139	3.7	11
13	3^3	79	11	3	-	-	3.7	-	41	37	137	73	3	7.11	13	3
15	5.11	3.5	5	5	3.5	5.7	5	3^2.5	5.13	5.17	3.5	5.11	5.7	3.5	23	47
17	-	29	3.13	-	7.19	3^2	37	-	23	11	17	3.7	53	-	3^3	13
19	3	131	23	3.7	-	-	3	31	-	3.13	7	71	29	-	109	3.19
21	-	3^3	7	37	3	11	-	3.19	73	7	3^2	-	17	3	47	-
23	-	7	31	11	43	29	-	-	3^2	-	13	3	41	23	3.11	61
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
27	-	23	19	-	3^2	-	7	3	113	-	3.11	13	-	3.7	59	17
29	-	59	3^3	109	79	3.7	-	-	3.11	29	31	3^2	7	19	53	-
31	3.17	-	67	3	7	19	3.11	41	-	3^3	-	7	23	-	37	3
33	-	3.17	29	7	3.11	31	23	3^2	-	-	3.7	-	47	3	83	11
35	5.13	5	5.7	5	5	3^2.5	5	5	5.19	5.7	61	3.5	5	5.11	3^2.5	53
37	3.11	7	73	3^3	83	-	3	13	59	3	107	11	3^2	89	67	3.7
39	7	3.13	41	-	3.17	127	29	3.7	37	43	3^3	23	-	31	7.13	-
41	-	-	3	19	-	3.17	7	11	3.13	-	-	41	-	7	3	43
43	3	-	13	3	-	37	3.17	-	31	3	-	-	3.7	-	19	3.13
45	5	3.5	5	5.11	5.7	5	5.19	5.17	5	5.13	29	5.7	5	3^2.5	5.11	59
47	-	11	37	31	89	61	-	-	3.17	-	23	3^3	11	-	3	-
49	3^2	113	7	29	23	-	41	-	-	3.7	11.13	-	3	-	-	3
51	53	3.7	43	-	3.13	71	-	3^2	7.11	47	3.17	-	107	3	29	41
53	7	-	3	-	-	3^3	11	7	43	-	113	3.17	19	-	3.7	23
55	3.5	5	5	3^2.5	5.11	5.13	5.7	29	5	23	5	5	5.17	5.7	43	5.11
57	-	41	11	23	3	7	31	3	47	-	3^2	61	7.13	3.17	-	19
59	29	-	3	-	7	3	13	19	3^2	-	41	3.7	73	-	3.17	-
61	3.13	31	-	3.7	-	-	3^3	-	-	3.11	37	29	71	13	23	3.17
63	-	3	53	-	3^2	-	-	3.11	23	7	3.19	-	-	3^3	31	-
65	5.17	5.7	5.19	59	29	5.11	5	37	5.7	5	5	3^2.5	5	5	5.13	5.7
67	3^3	17	71	3.11	-	41	3	43	13	31	97	131	3	19	7.11	29
69	-	3.11	89	53	37	19	47	3^3	-	-	3	67	11	3.7	41	13
71	-	-	79	17	31	3.7	-	23	29	13	11	3	7	-	3^3	67
73	3	23	103	3.13	7.17	-	3	-	97	3	59	7	3^2	-	-	3
75	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	-	-	3.7	-	139	3	17	-	3^3	41	-	3.19	23	79	3	11
79	43	7	11	3.19	103	-	23	17	7	3	-	13	61	11	-	3^2
81	23	61	-	131	3^2	29	43	3.7	17	89	3	11	-	3^2	7.19	-
83	-	-	3^9	73	59	3	7.19	-	3	109	-	3^2	37	7	3	-
85	3^2	5	31	3.5	41	5.7	5.13	5.11	5	3^2.5	5.17	23	5.7	5	5	3.5
87	13	3	-	47	3.7	23	53	3^2	-	29	3	7.17	137	41	-	31
89	-	19	3	7.11	-	3^2	-	13	3	-	7	3	17	-	3.11	139
91	73	137	29	3^3	-	-	37	61	103	3.7	31	59	3.11	17	13	3
93	101	3.7	47	-	3.19	-	71	53	7.13	-	3^3	-	-	29	17	7
95	5.7	5	5.13	37	23	31	5	5.7	5.11	5	5	3.5	5	5	5.7	5.13
97	67	-	-	3	101	-	3.7	19	-	3.13	103	43	3	7	-	3^2
99	31	47	-	13	3^3	7	101	3	53	-	3	-	7	3^2	-	23

A
COMMERCIAL DICTIONARY;

CONTAINING

DEFINITIONS AND EXPLANATIONS OF THE PRINCIPAL COMMERCIAL TERMS;

AND COMPRISING MUCH

MERCANTILE INFORMATION.

[Compiled principally from Nesbit's Arithmetic.]

A.

Abandonment. Its most ordinary application in commercial transactions is to marine insurances. It then signifies the exercise of a right which the *Assured* claims, to call upon underwriters or insurers to accept of what is saved, and to pay the full amount of the insurance, as if a total loss had happened.

Abatement, is a discount allowed for prompt payment. It is likewise allowed at the Custom House on goods which have received damage by salt water, &c.

Abbreviations, the initials, or characters, of commercial terms, used in marking bales, casks, or other packages of goods.

Acceptance, is an engagement to pay a bill, when due, according to the tenor of the acceptance.

Accommodation Bills or **Notes**, are those to which the *Drawee* only lends his name, and the *Drawer* engages to furnish him with the means of payment when the bills fall due.

Account-Current, is a statement of mercantile transactions with correspondents, drawn out in a plain and circumstantial manner, in the form of *Dr.* and *Cr.*, showing how affairs stand between the parties at the time when the account is made up.

Account-Sales, is an account of goods sold on commission, drawn out by the agent to whom they were consigned, to be sent to his employer, or the person who made the consignment.

Act of Honor, an instrument drawn up by a Notary Public, when a bill is accepted for the honor of another person.

Action, is, in a mercantile sense, applied to a law proceeding, instituted to recover payment of a debt due, and unpaid.

Acquittance, a discharge in writing acknowledging payment of a debt.

Adjustment, the settling averages on losses, or policies of insurance on ships or merchandise.

Administrator, a person intrusted with the administration of the goods of a person deceased, in default of an executor.

Advance, money paid before goods are delivered, or in case of consignment, before they are sold.

Advice, mercantile intelligence. To advice a bill, is to describe the amount, date, term, to whom payable, &c., and request the person on whom it is drawn to accept it.

Affreightment, the act of loading a vessel with goods.

Affidavit, signifies an oath or affirmation in writing, sworn before some person who is authorized to take the same.

Agent, a person employed, or taken to be employed by another, to transact business *in the place* of his employer.

Agio, a bank term in Hamburg, expressing the difference between bank money, or current money, and cash.

Allotting, or **Allotment of Goods**, is when a ship's cargo is divided into several parts, bought by different persons, whose names are written on as many pieces of paper, which are applied by a neutral person to the several lots or parcels, by which means the goods are divided impartially.

Allowance, a customary deduction in weights and measures, such as *draft*, *tare*, *ullage*, &c.

Alloy, a baser metal mixed with a finer one. In the United States coins, both the gold and the silver contain $\frac{9}{10}$ pure metal, and $\frac{1}{10}$ alloy. The alloy of gold is frequently estimated by carats, that of silver by pennyweights. In England, the standard for gold coin is 22 *carats fine;* that is, 22 parts of pure gold, and 2 of alloy. The standard for silver is 11oz. 2dwt. of pure silver to 18dwt. of alloy, making together 1 lb. Troy.

Annuity, a periodical payment of money, either yearly, half-yearly, quarterly, weekly, or at any other intervals.*

Annul, in Book-keeping, signifies to render an article of no import in the account. Instead of *erasing a sum* which has been entered by mistake on the *Cr.* side of an account, the account is *debited* for the same sum; and, on the contrary, if a sum has been entered on the *Dr.* side, the same sum is placed to the *Cr.* side of the account.

Antedate, a spurious or false date, prior to the true date of a bond, bill, or letter.

Appraiser, a person licensed to set a value upon goods to be sold.

Arbiter, or **Arbitrator**, a judge appointed by the parties, to whose decision they voluntarily submit.

Arbitration, an amicable way of settling disputes, by referring them to the decision of persons chosen by the parties.

Arbitration of Exchanges, is a comparison made between the exchanges of different places, for the purpose of negotiating bills to advantage.

Assay, in *a technical sense*, means the process by which the fineness of coins or bullion is determined.

Assignee, a person appointed to manage the affairs of a bankrupt. In some places, the same person is called a *Trustee.*

Assurance, *vide* **Insurance**.

Attachment, the act by which a Creditor may claim and seize the effects of his Debtor, wherever they may be found.

Average, a contribution made for losses at sea. Averages are distinguished into general and particular.

General Average, is a proportionable contribution paid by all the proprietors of a ship and cargo for losses, which are made with a view to safety, such as throwing goods overboard, or cutting away masts to prevent shipwreck.

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* The literal and original meaning of "Annuity," (from *annus*, Lat., a year,) is, a *yearly payment*, or a payment to be made *once in a year;* but usage has enlarged the signification of the word, so as to include *periodical* payments, though made at shorter intervals.

Particular Average is a contribution for such damages or losses as may happen from the common accidents of sea.

Award, is the judgment of arbitration of one or more persons, at the request of two parties who are at variance, for ending the matter in dispute.

B.

Bailment, is a delivery of things, such as writings, goods, &c., in trust, on condition that they shall be returned when the terms of the agreement are fulfilled.

Balance, the difference between the *Dr.* and *Cr.* sides of an account in a merchant's books.

Balance *of Trade*, the difference between the commercial exports and imports of one country with respect to another.

Bale, a pack, or quantity of goods, or merchandise.

Ballast, a quantity of stones, gravel, sand, or any other heavy material, laid in a ship's hold, in order to make her draw the more water, and render her stiffer, or more capable of carrying sail, without the danger of oversetting.

Balliage, a small duty paid to the city of London by aliens and others, for certain commodities exported by them.

Banco, bank-money; particularly applied to banks of deposit. At Hamburg it is used to distinguish *banco-money* from *current-money*.

Bank, a common repository intended to serve for the safe custody of money, and to facilitate its payment from one individual to another.

Bank-Bill, a note signed by one or more of the officers of a bank, promising to pay the sum therein mentioned, at an appointed time.

Bank-Note, a promissory note issued from a bank, payable on demand. The terms Bank-Bill and Bank-Note are often used as synonymous.

Banker, a proprietor of a banking-house; or the trustee for a merchant or other person, who is possessed of money which he does not wish to retain in his own hands, but places it for security, with some banker, from whom he draws it in such sums and at such times as his occasions require.

Bankrupt, an insolvent person, or one who is unable to pay his debts.

Barratry, any fraud committed by the master or crew of a ship, on the owners or insurers; such as sinking, deserting, or taking away the ship, or embezzling the cargo.

Barter, the exchanging of one commodity for another. Such was the original mode of Commerce till money was invented.

Bill, a term generally applied to a draft or promissory note, sometimes to an advertisement, and frequently to an account of goods.

Bill of Entry, a note or list of the particulars of goods entered at the Custom House.

Bill of Exchange, a written order for the payment of money, issued from one place and directed to another.

Bills of Exchequer, are those issued by the English government for different sums, which bear interest until paid off, at the rate of 2½d. or 1½d. ⅌ £100 ⅌ day. They are sometimes at a premium, and sometimes at a discount, as they fluctuate with the Public Funds.

Bill of Health, an account of the health of the crew, given in by the captain or master of a ship.

Bills, India, bills drawn in India on the East India Company in London, and payable at the India House.

Bill of Lading, is an acknowledgment signed by the master of a ship, and delivered to the shipper or proprietor of the goods. It contains an account of the articles received on board, and a promise to deliver them at the intended place for a specified sum. Three bills of lading are usually made out, one for the shipper, another to be held by the captain, and the other to be sent to the person to whom the goods are addressed.

Bills, Navy, are those issued by the British Navy Board, in payment of stores for the ships, dock yards, &c. They are made payable at 90 days, with interest at 2½d. ⅌ day on each £100.

Bill of Parcels, a note of the quantity and value of goods sold, given by the seller to the buyer along with the goods.

Bill of Sale, a sealed contract by which the right or interest in certain goods is conveyed from one person to another.

Bill of Store, a license granted by the British Custom House to merchants, to carry such stores or provisions as are necessary for a voyage, free of duty.

Bill of Sufferance, a license granted to a Merchant at the Custom House, in Great Britain, allowing him to trade from one port to another, without paying Custom.

Bills, Victualling, are those issued by the British Victualling Board, like Navy Bills by the Navy Board.

Blank Credit, the permission which one house gives to another to draw on it to a certain amount, at any time, for their accommodation.

Blank Letter of Attorney, one in which a *void space* is left to be filled up with the name of the person who is to act.

Blank Endorsement *of Bills of Exchange*, is when the holder only writes his name on the back of a bill, leaving an empty space sufficient to write an order or a receipt.

Bond or Obligation, a deed whereby a person binds himself, his heirs and executors, to pay a certain sum, or do some other act as therein specified.

Bonds, India, are those issued by the East India Company for £100 and £300 each, bearing interest at 3½ ⅌ cent. ⅌ annum.

Bond, post Obit, a bond which only becomes payable after the death of some person whose name is therein specified.

Bonded Goods, are certain articles, which on being landed, are warehoused, upon a bond being given by the Owner for payment of the duties, &c.

Book of Cargo, *or Loading*, a book kept by the Mate of a trading vessel, containing the particulars of the goods on board.

Book of Rates, a book showing the duty to be paid at the Custom House, for goods exported and imported.

Bottomry, the act of lending money on a ship's keel, or bottom. The lender advances money to the owner of a ship, who pledges his property in the same as security; and engages to repay the money with a stipulated advance, when the ship returns; but to pay neither loan nor interest if the ship be lost.

Bounty, a premium given for the encouragement of some branch of trade, manufacture, or agriculture.

Broker, a person employed by Merchants in buying and selling;

who, for a small ℘ Centage, finds his Employer Buyers in the one case, and Sellers in the other. There are Ship Brokers, Insurance Brokers, Exchange Brokers, Stock Brokers, &c.

Brokerage, the allowance paid by Merchants to Brokers, either for the sale or purchase of Goods, Bills of Exchange, Stock, &c.

Bullion, uncoined gold or silver in the mass.

C.

Cambist, a dealer in Bills of Exchange, or a person skilled in the business of Exchanges.

Capital, among Merchants, Bankers, and Traders, is the sum which they put into Trade.

Carat, a word used in expressing the fineness of gold. *Vide* Alloy.

Cashier, he who keeps the *Cash* or the Money, which it is his business to receive and pay.

Certificate, a paper giving certain information of anything, and signed by persons competent to judge of the accuracy of its contents.

Charter, a deed by which the Legislature confers any grant or privilege on one person or more, or on any body politic.

Charter-Party, a contract executed by the Freighter, and the Owner of a Ship, containing the terms upon which the ship is chartered or hired for the voyage.

Chattels, all kinds of goods and property, movable or immovable; except freehold property.

Check, a draft on a Banker, made payable to the Bearer.

Chevisance, a composition between Debtor and Creditor.

Circulating Medium, Cash, or Bank-Notes or other paper money payable on demand.

Circular Letter, the printed notice of the establishment of a Mercantile-house, or alteration in the *Firm*, &c.

Clearing *a Vessel*, is entering all particulars relating to her, at the Custom-House.

Clearing of **Drafts**, *or Checks*, the mode used by City Bankers for exchanging Checks on each others' houses and settling the differences.

Cocket, a Custom-House warrant, given on the entry of goods for exportation, to signify that they have paid the duty.

Commerce, the exchange of commodities for other articles, or for some representative value, for which other commodities can be procured; in other words, *Mercantile Business in General.*

Commission, an allowance of so much ⅌ Cent., given to Agents or Factors, for transacting the business of others.

Commission of Bankruptcy, (in England,) an order under the great seal, directing five or more Commissioners to inquire into the state of the Bankrupt's affairs.

Commissioner, a person who has a commission, as letters patent, or other lawful warrant, to execute any public office.

Company, persons united in trade or commerce.

Composition, a contract between an insolvent Debtor, and his Creditors, by which the latter accept of a *part of their debts* in compensation for the whole.

Compromise, to adjust a difference or dispute by mutual concession.

Consignee, an Agent or Factor to whom goods are sent to be sold on Commission.

Consignment, goods sent by one House to another to sell on their account, allowing them a certain commission ⅌ Cent.

Consul, an officer established by virtue of a commission from the King or other Rulers, in all Foreign countries of any considerable trade, to take care of the affairs of commerce of his own nation.

Contingent, the proportion which falls to the share of a person concerned in any business or adventure.

Contraband Trade, a prohibited commodity of Merchandise, bought or sold, imported or exported, in prejudice of the laws and ordinances of a state, or the public prohibitions of the Sovereign, or of the law of nations.

Contract, a covenant or agreement between two or more persons.

Contribution. *Vide* Average.

Convoy, one or more ships of war, employed to accompany and protect merchant-ships, and prevent their being insulted by pirates, or enemies of the State, in time of War.

Counter-Order, an order sent to revoke a former one, either for the sale or purchase of any commodity.

Countervailing Duties, equal duties established between two countries, and charged on the exportation and importation of the same kind of goods.

Cranage, money paid for the use of a crane. A Crane, is a machine used on Wharfs and in Warehouses for raising and lowering goods, &c.

Credit, in commerce, a trust or loan of Merchandise or Money.

Creditor, a person to whom any sum of money is due, either by obligation or otherwise.

Credit Side, the right-hand page or side of an account.

Currency, money in common circulation, whether coin or paper. In America and the West Indies, money of account is called currency, being used instead of *sterling*.

Custom, a tax upon Goods exported or imported.

Custom-House, the place where entries are made of goods exported or imported, and the duties paid; or where drawbacks are allowed on goods exported, and the documents are received by which the money allowed is to be obtained.

D.

Day-Book, the Book in which are entered the particulars of the Sales, and sometimes the amount of the Purchases of Goods, with other occurrences in Business.

Days of Grace, are a customary number of days allowed for the payment of a Bill, after the same becomes due.

Debenture, is a certificate delivered at the Custom-House, when the Exporter of Goods or Merchandise has complied with the proper regulations, by which certificate he is enabled to receive a bounty or drawback on the exportation.

Debit, a term used in Book-keeping to express all articles supplied on the subject of an account, or charged to that account.

Debit Side, the left-hand page or side of an account.

Debtor, a person owing money to another, who is his creditor.

Deed, an instrument in writing containing some conveyance, contract, bargain, or agreement.

Del-Credere, a charge made for the guarantee of debts. It is sometimes called *Guarantee.*

Demand, calling upon a man for any sum of money, or any other thing due. A Bill at sight, therefore, is payable on demand.

Demurrage, is an allowance made to the Master of a Ship by the Merchants, for being detained in port longer than the time agreed upon.

Deposit, a sum advanced in part payment, and which is to be accounted for in the general balance; a security to perform a duty, to be restored when such duty is performed; any article intrusted to others, for use, or for safe keeping.

Depot, a place in which Stores are deposited, for an Army.

Deviation, is a departure without necessary cause, from the regular course of a voyage. This deviation makes void the contract of Insurance.

Discount, is an allowance made by the seller of Goods to the Buyer, when he pays *cash*, instead of taking the *credit.* The usual allowance for cash is 5 ℔ Cent.

Dishonor, a term used when Bills of Exchange, &c., are refused acceptance or payment.

Dividend, a share of any capital, debt, or profit; also a share or proportion of the interest of Stocks or Public Funds, divided among and paid to the proprietors.

Dock, a place where Vessels are built, repaired, laid up, or lodged for loading or unloading.

Docket, a short memorandum or summary affixed to larger papers. *Striking a Docket*, is when a Creditor gives bond, proving his Debtor to be a Bankrupt.

Draft, a Bill or check by which one person draws money on another. Also a small allowance on goods sold by weight.

Drawback, certain duties, either of the Customs or of the Excise, allowed upon the exportation of some of our manufactures, or upon certain merchandise, that have paid inward duties.

Drawee, the person on whom Bills of Exchange are drawn.

Dubious Paper, means Bills drawn on Houses of little Credit.

Due Protection, signifies regular acceptance or payment of a Draft or Bill.

Duty, the impost laid on Merchandise at importation or exportation, commonly called the Duties of Customs.

E.

Earnest, is the money advanced to bind the parties to the performance of a verbal agreement.

Embargo, an arrest on Ships or Merchandise by Public Authority.

Embezzlement, the act of appropriating that which is received in trust for another.

Emporium, a principal place for the importation and sale of Merchandise.

Endorsement. *Vide* Indorsement.

Engrossing, purchasing a large quantity of any commodity in order to raise the price.

Entrepot, a public Magazine in most foreign countries, for the reception of Merchandise imported.

Entry, a statement of goods imported or exported, as entered in the Custom-House Books.

Exchange, a place in most Large Cities where Merchants, Agents, Bankers, Brokers, and other Commercial Characters meet, to confer and treat together of matters relating to Exchanges, Remittances, Payments, Adventures, Assurance, Freights, and other Mercantile Negotiations both by Sea and Land.

Exchanges, the paying or receiving of money in one country for its equivalent in the money of another country, by means of Bills of Exchange.

Exchequer, the Court to which all Revenues belonging to the Crown are brought.

Excise, an inland tax on articles manufactured or consumed, as malt, wine, and other liquors.

Exportation, the act of sending Goods out of one country into another.

F.

Factor, an Agent or Correspondent residing at a distance, and commissioned by Merchants to buy or sell goods on their account. *Vide* Agent.

Factorage, the allowance or commission given to Factors or Agents by Merchants or Manufacturers who employ them.

Factory, is a place where a considerable number of Merchants and Factors reside, to negotiate business for themselves and Correspondents on Commission.

Failure, is where a person in trade, through misfortune, or imprudence, is unable to pay his debts.

Finances, a term usually applied to the Public Revenues.

Firm, the mercantile appellation of a House engaged in Commerce.

First-Rate Paper, bills drawn or accepted by a good House, such as has always paid its bills regularly.

Flat. An article of Merchandise is said to be flat, when there are few Buyers.

Flotsam, is when a ship is sunk or cast away, and the goods float on the sea.

Fore-Closing a mortgage, is to compel payment, or dispose of the property.

Fore-Stalling, the purchasing and laying up goods before they come to market, with the intention of raising the price.

Folio, the debtor and creditor side of a Ledger or Account-Book.

Freight, the goods with which a ship is loaded. Also, the money paid for the use or hire of the vessel.

Fund, a stock or capital by which any expense is supported.

Funds, *Public*. *Vide* **Stocks**.

Fuori *Banco*, out of the Bank; the current money of Genoa.

G.

Garble, the dust, dross, and refuse of spices and drugs.

Garbling, the picking out the worst of any commodity.

Gauger, an officer appointed to gauge or ascertain the contents of any excisable commodity.

Gazette, a paper published by the English Government, containing, among other things, notices of the dissolution of Partnerships, Commissions of Bankruptcy, Suspension or Continuance of Bounties, Embargoes, &c.

Government, "for your government," signifies to give your Agent or Correspondent "better information and rules for acting by," in the purchase or sale of merchandise.

Gross, signifies twelve dozen of any article.

Gross Weight, the whole weight of merchandise with the dust and dross; as also the bag, cask, or chest in which the goods are contained.

Groundage, a small duty payable in certain ports by ships coming to anchor.

Guarantee, a person who undertakes that certain stipulations shall be fulfilled.

Guild, a company or society of men incorporated.

H.

Hallage, money paid for haling, drawing, or carrying goods to and from ships. Also, the toll due to the Lord of a fair or market, for such goods as are sold in the common hall of the place.

Honor, to honor a draft is to accept it on being presented.

Husbandage, the managing owner's (of a ship) allowance or commission.

I.

Importation, the act of bringing goods into a country from foreign parts.

Impost, a certain tax levied on goods imported.

Indemnity, is where one person secures another from responsibility against any particular event.

Indorsement, the signature which the holder of a negotiable bill writes on the back of it, by which he transfers his interest to a third person.

Ingrossing, buying up large quantities of corn or other provisions, with a view to raise their price, and sell them again.

Insolvent, a person in trade who has not a capital adequate to the payment of his debts.

Instalments, certain proportions in which, by agreement between debtor and creditor, a sum due is to be paid at stipulated times.

INSURANCE *Broker*, a person employed by merchants to effect insurance on their ships and cargoes.

INSURANCE or **ASSURANCE**, is a contract of indemnity, by which one party engages, for a stipulated sum, to insure another party against a risk to which he is exposed. The *Insurer* or *Underwriter* is the party who takes upon him the risk; and the *Insured* is the party protected by the Insurance. The sum paid is called the *Premium*, and the Paper containing the contract the *Policy*.

INTEREST, a premium paid for the use or loan of money. *See* USURY. "To guard a person's interest," is to protect his property, and watch over his concerns.

INVENTORY, an account or catalogue of effects; a schedule.

INVOICE, a paper sent off with goods exported on commission, or for the shipper's own account.

J.

JERQUING, of a ship, is a search performed by an officer of the Customs (called *Jerquer*) after she is unloaded, to see if there are no unentered goods concealed.

JETSAM, is when a ship is in danger of being sunk; and, to lighten her, the goods are thrown into the sea, and the ship notwithstanding perishes.

JOURNAL, a Book in which are recorded the transactions of the *Day-Book*, and the subsidiary Books; having the Drs. and Crs. pointed out, in order to post them with more ease into the Ledger.

K.

KAY, KEY, QUAY, a wharf or structure by the water side, in a seaport, for loading and unloading goods.

KEYAGE, the money or toll paid for loading or unloading goods or merchandise at a key or wharf.

L.

LAGAN, is when goods are cast into the sea to lighten a ship, and which are so heavy that they sink to the bottom, and therefore the mariners fasten to them a buoy or cork, so that they can be found again.

Land-Waiter, an officer of the Custom-House, whose duty it is, upon the landing of merchandise, to take an account of the same.

Lastage, the ballast or lading of a ship. Sometimes the word is used for garbage, rubbish, &c.

Leakage, an allowance in the Customs granted to the importers of liquors which are supposed to have received damage in their passage.

Lease, a contract by which, in consideration of some payment, a temporary possession is granted of houses or lands.

Ledger, is the Principal Book of accounts kept by merchants and tradesmen, in which the state of every person's account is seen.

Legacy, a bequest or gift of money, goods, or chattels by testament.

Legatee, the person to whom a legacy is bequeathed.

Letter of **Advice**, a letter giving notice of any transaction, such as advising your correspondent that you have drawn on him, shipped goods to his order, &c.

Letter, or **Power** of **Attorney**, a writing which empowers one person therein named to act for another.

Letter of **Credit**, is where a merchant or correspondent writes a letter to another, requesting him to credit the bearer with a certain sum of money.

Letter of **License**, is a written permission granted to a person under embarrassment, allowing him to conduct his affairs for a certain time, without being molested.

Letters of **Marque**, are commissions granted to captains or merchants in time of War, to make reprisals on the ships of the enemy, on pretence of indemnification for injuries received.

Letters Patent, a privilege granted to an inventor, or discoverer, to entitle him exclusively to enjoy the advantages of his invention, or discovery, for a term of years.

License, a legal permission to carry on some branch of business on which a duty is laid.

Lien, a claim or attachment on any property which a person has in his possession, for a debt due to him from the owner of the property.

Life Annuities, annual payments to continue during any given life or lives.

Light Bills, charges paid to the Trinity House, London, for Lighthouses, Buoys, &c.

Light-House, a tower situated on an eminence upon the sea coast, or at the entrance of some port or river, for the direction of ships in dark nights, by means of some sort of illumination, as fire, candles, or lamps.

Lighterage, money given for loading or unloading a ship, by means of a boat or lighter.

Liquidation, the winding up of a business, such as paying and receiving all debts relating to the concern.

Loan, anything lent on condition of being returned; or sums, generally of large amount, borrowed from individuals or public bodies, for the service of the state, and which form part of the National Debt. *See* Stocks.

Lot, a portion or parcel of goods. This term is usually applied to any parcel of goods put up at once, at an auction or public sale.

M.

Manifest, a regular list of a ship's cargo, containing the mark and No. of each separate package, and the names of the persons to whom they are consigned, which paper must be signed by the master of the vessel, before any of the goods can be landed.

Manufacture, an article produced by labor or machinery, from any raw material.

Manufactory, a place where several artificers are making any commodity, or article of merchandise.

Mart, a place of public traffic; a great market or fair.

Maturity, in Bills, is when they become due, or payable.

Maximum, the highest price of any article, as fixed by some law or regulation.

Merchandise, all sorts of goods which may be bought or sold.

Merchant, a wholesale dealer in any kind of goods, on his own account.

Minimum, the lowest price of any article as fixed by some regulation.

Mint, the place where the public or current money is coined.

Money, metal coined for the purposes of commerce.

Monopoly, the sole power or privilege of selling any commodity, whereby any person, or bodies politic or corporate, are sought to be restrained of any freedom they had before.

Mortgage, a pawn of land, or tenement, or anything movable, laid or bound for money borrowed. The person borrowing the money is called the *Mortgager*, and the lender the *Mortgagee.*

Mulcts, fines laid on ships or goods for the maintenance of Consuls, Garrisons, &c.

Mutual Debts, when two traders are indebted to each other, one debt may be set off against the other; and in case of bankruptcy, mutual credits, as well as mutual debts, may be set off.

Mutual Promise, is when one person promises to another to pay money, or do some other act; and the latter, in consideration thereof, promises to do a certain act, &c.

N.

National Debt, is a debt due by any whole Political Community, as represented by their Legislature and Government.

Navy Bills. *See* Bills, Navy.

Net Proceeds, the sum which goods produce, after every deduction is made.

Net Weight, the weight of any commodity, after every deduction is made, and for which the price is charged.

Non-Claim, is where a creditor neglects to make his claim within a proper time, in which case he cannot enforce his demand.

Notary-Public, a person duly appointed to attest deeds and writings; he also notes or protests bills of exchange, inland and foreign, and promissory notes, when refused or returned.

Note, a memorandum, or a written order for money.

Noting, is the act of a Notary, when a bill or draft is not duly honored; or, in other words, refused acceptance or payment.

O.

Obligation, a bond containing a penalty with a condition annexed, either for the payment of money, performance of covenants, or the like.

Obligee, he to whom a bond is made payable.

Obligor, the person who enters into a bond, or he by whom it is to be paid.

Office, a place where business is transacted.

Omnium, a term used among English Stock-jobbers, to express all the articles included in the contracts between Government and the original Subscribers to a Loan.

Ordnance Debentures, Bills issued by the Board of Ordnance for the payment of Stores, &c., purchased for that department.

Order, a direction from one house to another, to effect certain Purchases, &c., upon limited or unlimited conditions.

P.

Pack or **Package**, any quantity of goods tied up for carriage. The latter term also denotes the charge made for tying up the goods.

Packer, a person who carries on the business of packing goods.

Par of Exchange, is the intrinsic value of the money of one country, compared with that of another, including weight, fineness, &c. [It is affected by the balance of trade.]

Parcel, a term applied both to small packages of wares and to large lots of goods. In this last sense, 20hhds. of sugar, or more, if bought at one price, or in a single lot, are denominated "a parcel of sugar."

Partnership, is when two or more persons unite in trade, and agree to participate in the profits or losses, according to their respective shares in the capital, or attention devoted to, the Concern.

Part-Owners, persons concerned in ship-matters, and who have joint shares therein.

Pass in Conformity, or "*to state in conformity*," is to acknowledge that an account transmitted is correct.

Pawn, a pledge lodged for the security of the payment of a sum of money borrowed.

Pawnbroker, a person who advances money upon pledges, for which he is allowed interest, at different Rates, according to the Sums advanced upon the Goods.

Payee, the person to whom a bill is made payable; and also the person entitled to receive payment of any sum of money.

Payer, he who binds himself to the payment of bills, bonds, or any sum of money, either verbally or in writing.

Penalty, a forfeiture for disobedience to certain laws or regulations. A penalty also is often annexed to secure payment of money, the performance of certain covenants in a deed, articles of copartnership, &c.

Permit, a license from the Excise, for persons to remove excisable articles, denoting that the duties have been paid.

Perpetuity, the number of years' purchase to be given for an annuity which is to continue for ever.

Pesage, money paid for weighing goods or any merchandise.

Pierage, money paid by masters of ships for the use of a pier.

Pilot, a person employed to conduct ships over bars and sands, or through intricate channels, into a road or harbor.

Pilotage, money paid for piloting a ship.

Policy of Assurance, the deed or instrument by which a contract of assurance is effected.

Portage, money paid for sailors' wages while in port; also, money paid for the use of a port in shipping or landing goods.

Port-Sale, a sale of goods upon the key.

Post-Entry. When goods are weighed or measured, and the merchant has got an account thereof at the Custom House, and finds his entry already too small, he must make a *post* or an additional entry for the Surplusage.

Posting, in *Book-keeping*, is the mode of transferring articles from the Journal, or the Subsidiary Books, to the Ledger.

Poundage. the customs and other duties, payable on all goods imported, (except those free of duty or liable to tonnage;) being in most cases levied at so much in the *pound*, or ℔ Cent., on the respective values.

Pre-Emption, a first buying, or the right or privilege of buying before others.

Premium, the money paid to an underwriter, for insuring the safety of ships, goods, houses, &c.

Price-Current, a list of the various articles of merchandise in the market, with the present prices annexed to each. In most of the great commercial cities and towns, lists of this description are generally published once or twice a week.

Primage, a certain allowance paid by the shipper or consignee of goods, to the mariners and master of a vessel for loading the same.

Principal, the capital sum due, or lent, in opposition to interest. It also means the head of a *Firm*, or Commercial House.

Prisage, is that custom or share, that belongs to the King, out of such merchandise as is taken at sea, by way of lawful prize.

Procuration, is where a merchant empowers his clerk or agent to write his Signature or *Firm*, in accepting or endorsing bills, &c.

Pro Forma, a term generally applied to a paper showing the terms or charges attending any particular business, as *Pro Forma Account Sales*, &c.

Promissory Note, a note of hand promising the payment of a certain sum, at a stated period.

Protest, a paper made out by a Notary Public, declaring that a bill has been presented for acceptance or payment, and was refused.

Q.

Quarantine, the time that a ship suspected of infection, is restricted from intercourse with the shore; it also means the duty imposed on ships for the purpose of *Quarantine*.

Quay, or **Key**. *See* **Kay**.

Quest-Men, an English term for persons appointed to inquire into abuses, especially such as relate to weights and measures.

Quotation, a term generally applied to a list of the Prices of Exchange.

Quoted on Board, means the price for which a merchant agrees to put goods on board, free of expenses of shipping, to the buyer.

R.

Re-Assurance, a contract which the first assurer enters into, in order to relieve himself from the engagements which he has incautiously undertaken, by throwing them upon other underwriters, who are called re-assurers.

Rebate, is an allowance in the purchase of goods, for prompt payment. *See* Discount.

Receipt, an acknowledgment in writing of having received a sum of money, or other value; and is a voucher either for an obligation discharged, or one incurred.

Re-Exchange, or **Re-change**, the Price of a new Exchange, due upon a bill that has been protested. It must be refunded by the drawer or endorser.

Refraction, is a deduction from the rate of goods for damages.

Refunding, the paying back of the money received in consideration of a contract, the conditions of which have not been fulfilled.

Regrating, buying any wares or victuals, and selling them in the same market or fair.

Release, is the relinquishment of any right of action, which a man has, or claims, against another.

Remittance, a sum of money sent either in bills of exchange or otherwise, from one house to another.

Renewal of a Bill, is the cancelling a bill or promissory note due, and accepting another at a certain date in lieu thereof.

Respondentia, a contract on which money is borrowed on the security of goods and merchandise, the same as in Bottomry on the security of a ship.

Restitution, when money has been paid wrongfully or by mistake, is the right which the person so paying has to demand it back.

Retail, is a dealing in commodities in small quantities.

Returns, a term expressing the value, in goods or in money, returned by the consignee of a cargo or parcel of goods to the consignor. The term also means, a return or remittance of bills.

Return of Premium, the whole or part of the premium of an insurance which is given back according to the terms of the *Policy*.

Revenue, signifies the produce arising from any possessions. Thus the public revenue of a kingdom is the produce of its imposts, in shape of rents, taxes, duties, &c.

S.

Salvage, an allowance made for saving a ship or its goods, or both, from the danger of the seas, fire, enemies, &c.

Sample, a small quantity of an article at a public or private sale, as a specimen of the commodity.

Schedule, in commerce, the statement of a Bankrupt's Affairs, delivered by him to the Commissioners appointed to investigate his Case. Also, a paper containing a list of goods.

Searcher, an Officer of the Custom-House, whose business is to search all ships outward bound, to see whether any prohibited or uncustomed goods are on board.

Sea-worthy, a term applied to a ship which is, in every respect, fitted for her destined voyage.

Seizure, an arrest of some merchandise, movable, or other matter, either in consequence of some law, or by express order of the Government.

Set-off of Mutual Debts, is where tradesmen are mutually indebted, one debt may be set against the other; and in case any action be brought, notice is to be given of the particular sum or debt intended to be *set off* against another.

Smuggling, the act of importing or exporting goods without payment of the Customs, or Excise Duties.

Solidity, the character which a Mercantile House bears as to property.

Solvent, a person in trade who is able to pay his debts.

Specie, coin as distinguished from paper money.

Standard, a weight or measure of admitted authority, by which others are adjusted. It is also taken for the rule of fineness at which gold and silver are fixed, to be coined.

Staple, a place of public sale, or a town or city having such a place.

Staple-Goods, such as are sold at a staple; or the principal produce of a country; and also goods not being liable to perish, as wood, lead, iron, &c.

Stock, a fund raised by a Commercial Company to be employed in trade; in Book-keeping it denotes the Owner, or Owners of the Books. Stock is a term likewise applied to the Capitals of the Bank of England, to the East India and South Sea Companies, &c.

Stocks, or Public Funds, are the debts of Government, for which interest is paid from revenues set apart for the purpose. The mode of raising supplies for the State, by borrowing money from individuals or public bodies, and levying taxes for the payment of the interest, is called the *Funding System;* and the loans thus raised constitute the *National Debt.*

Stock-Broker, a person employed to buy or sell shares in the Joint Stock of a Company, or in the Public Funds.

Stock-Jobber, a person who deals in the Public Funds on his own account.

Storage, warehouse rent.

Subsidy, an aid or tribute granted to the King or Queen, by a tax on grounds, lands, &c. It also means a foreign supply or grant.

Supercargo, a person employed by Merchants to go on a voyage, and oversee their cargo or lading, and dispose of it to the best advantage.

Surety, a person who becomes bound that another shall pay a certain debt, or perform a certain act.

Suttle, the weight of goods after *Tare* is allowed, and when farther deductions are to be made, such as *Draft* or *Tret.*

T.

Tally, a piece of wood to score any account or reckoning upon.

Tare, an allowance for the weight of the bag, box, cask, or other package in which the goods are packed up.

Tarif, or **Tariff**, a table or catalogue containing the names of different kinds of Merchandise, with the duties to be paid.

Tellers, clerks in public offices, who reckon, receive, and pay money.

Tide-Waiters, or **Tidesmen**, officers appointed by the Custom-House, to remain on board Merchants' vessels, while they have any customable goods on board.

Tonnage, a custom or impost due for Merchandise brought or carried in Tons from, or to, other nations after a certain rate ℘ Ton. It also means the burden or number of Tons which a ship carries.

Tontine, a scheme upon which annuities for lives are granted, with the benefit of Survivorship.

Traffic, a general term for Trade, Commerce, Barter, or Exchange.

Transfer, the act by which one party makes over or assigns his right, interest, or property to another.

Tret, an allowance on the weight of goods for waste, usually 4lbs. on every 104, or $\frac{1}{28}$th part of the suttle; but this allowance is now seldom given. It is allowed in the *Draft* or *Tare.*

Tronage, a custom or toll for weighing goods.

Trustee, a person who has an estate, or money, put or trusted in his hands, for the use of another.

Tunnage, an impost of so much ℔ tun of 252 gallons on liquors imported or exported.

U.

Ullage, signifies so much of the contents of a cask or other vessel, as it wants of being full.

Umpire. When two arbitrators cannot agree in settling a dispute, an appeal is made to a third person named, who is called an umpire, and whose decision is final.

Underwriter, is an Insurer who subscribes his name to Policies of Insurance on Ships, Merchandise, &c.

Usance, the usual term or time at which a Bill of Exchange is drawn.

Usury, consists in taking more than *Lawful Interest*, for the Loan of Money; which, in England, was fixed at *Five per Cent.* per Annum, by Statute 12 Anne, Chap. 16th, 1713, on Pain of forfeiting *Triple* the Value of the Principal, by taking a Higher Interest.—By the Act 2 & 3 Victoria, Chap. 37th, 1839, Bills of Exchange and Promissory Notes, not having more than *Twelve Months* to run, before they arrive at Maturity, and also Contracts for Loans of Money above £10, are exempt from the operation of the preceding Statute. For the rate of Interest in the several states of the Union, see note on page 101.

V.

Value. To value in a mercantile sense, is to draw a Bill; the words "value received," or "value in account," are also mentioned in every Bill of Exchange, although not essential to their validity.

Vellon, a money in which accounts are kept in many parts of Spain.

Vendee, the person who buys.

Vendor, the person who sells.

Vendue, an auction or a public sale.

Voucher, a document or paper proving that some payment has been made, or other transaction effected.

W.

Warehouse, a receptacle for Wares and Merchandise.

Warehoused or **Bonded Goods**, certain articles which, on being landed, are warehoused, upon bond being given for the payment of duties, &c.

Waste, loss on Merchandise, through heat, leakage, &c.

Wharf, a bank or quay for landing goods at, as also for shipping off from.

Wharfage, money paid for the use of a Wharf. *Vide* Keyage.

Wholesale, dealing in Large Quantities, or on a Large Scale.

Wreck, in the legal sense, a ship or goods, driven ashore. When goods are lost at sea, they are distinguished by the barbarous appellations of *flotsam*, *jetsam*, and *lagan*. [q. v.]

THE END.

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